Section 1.4 Math 141
Lagrange Multipliers and Constrained Optimization

Main ideas

Sometimes we want to optimize (maximize or minimize) a function when there is some constraint (restriction) on what values its input values (its variables) can be.

Sometimes we can simply solve for one variable in terms of the others within the constraint and substitute this into the objective (the function to maximize or minimize) and optimize this function as described in Section 3. But sometimes we can't. This is where the Method of Lagrange Multipliers comes in.

In the Method of Lagrange Multipliers, the basic steps are:
1. Create the auxiliary function $F(x,y,\lambda) = f(x,y) + \lambda g(x,y)$ from the objective $f(x,y)$ and the constraint $g(x,y)$ functions.
2. Find $\frac{\partial F}{\partial x}$, $\frac{\partial F}{\partial y}$ and $\frac{\partial F}{\partial \lambda}$.
3. Solve for λ in both $\frac{\partial F}{\partial x} = 0$ and $\frac{\partial F}{\partial y} = 0$.
4. Set these two values of λ equal and use them to find a relationship between x and y: solve for one variable in terms of the other.
5. Substitute this relationship of one of the variables in terms of the other into $\frac{\partial F}{\partial \lambda} = 0$ (which is simply the original constraint equation), and solve for that variable. Use that value to find the value of the other variable.
6. Bonus: the value of λ tells us how much the objective would change if the constraint value were increased by one unit.

When dealing with three (or more) variables x, y, z, the steps are essentially the same, but Step 4 in particular is more complex.

Day 1, In Class

1. I'll work a maximization problem in class using the Method of Lagrange Multipliers. I'll refer to the steps above as they come up in our problem.

2. We'll work the example again, except we'll reverse which quantity is fixed (the constraint) and which quantity we want to optimize.

Day 1, In Groups (if time)

3. You will work HW 1.4.17. Use the Method of Lagrange Multipliers.

Day 2, In Class

4. I will work HW 1.4.34.

Day 2, In Groups

5. You will work HW 1.3.31. (Or just start into the homework problems.)