Math 141 Spring 2024 **Midterm Exam 4** **March 29, 2024**

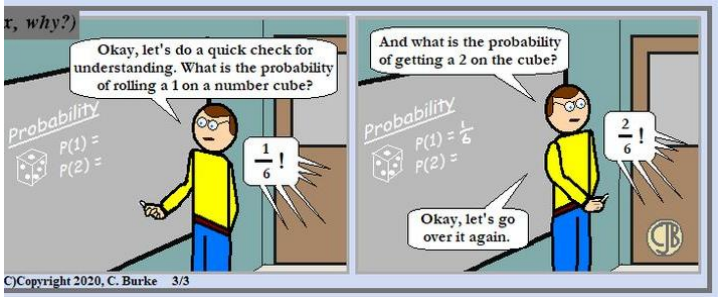
Name: Solutions

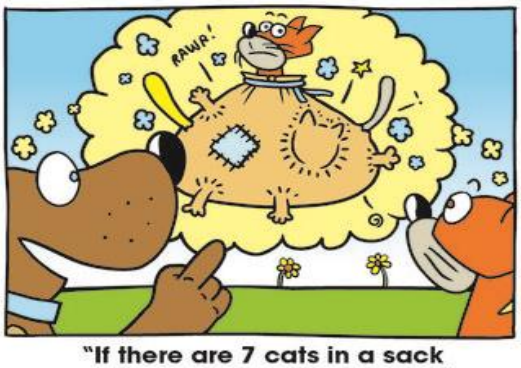
Problem	1 / 2 / 3	4 / 5	6	7 / 8	Total
Possible	31	30	26	13	100
Received					

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 x 5 card of notes, both sides, and a calculator.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.





"If there are 7 cats in a sack and I draw one at random,..."

"... what is the probability that I will draw you?"

You do **NOT** need to simplify your answers on this page.

14 points 1. Suppose we roll two 5-sided dice, numbered 1 to 5. We are interested in the sum.

Event E is "sum is ≥ 6"

Event F is "sum is odd"

Sum of 2 dice

	1	2	3	4	5
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10

/2 $\Pr(E) =$ $15/25$

/2 $\Pr(F) =$ $12/25$

/2 $\Pr(E \cap F) =$ $6/25$

/2 $\Pr(E \cup F) =$ $\frac{15+12-6}{25} = \frac{21}{25}$

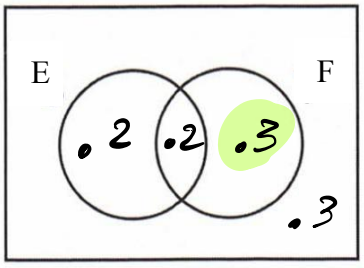
/2 $\Pr(E|F) =$ $\frac{6/25}{12/25} = \frac{6}{12}$

/2 $\Pr(F|E) =$ $\frac{6/25}{15/25} = \frac{6}{15}$

/2 Are events E and F independent? Explain why or why.

No. $\Pr(E|F) \neq \Pr(E)$, etc.

12 points 2. Suppose that $\Pr(E) = .4, \Pr(F) = .5, \Pr(E \cap F) = .2$.



Find each of the following.

/2 $\Pr(E \cup F) =$ .7

/2 $\Pr(E' \cap F) =$ .3

/3 $\Pr(F|E') =$ $\frac{\Pr(F \cap E')}{\Pr(E')} = \frac{.3}{.6} = .5$

/3 $\Pr(E'|F') =$ $\frac{\Pr(E' \cap F')}{\Pr(F')} = \frac{.3}{.5} = .6$

/2 Are events E and F independent? Explain why or why not.

Yes. $\Pr(F|E') = \Pr(F)$, etc.

5 points 3. Suppose 4 persons each randomly, independently choose a number between 1 and 20. What is the probability that 2 or more of their numbers match?

$= 1 - \Pr(\text{all diff.}) = 1 - \frac{20 \cdot 19 \cdot 18 \cdot 17}{20 \cdot 20 \cdot 20 \cdot 20}$

You do **NOT** need to simplify your answers on this page (except the last two problems).

15 points 4. Suppose that 10 standard (6-sided, with numbers 1 to 6) dice are rolled. What is the probability that you get:

/3 No 4's: $\left(\frac{5}{6}\right)^{10}$

/5 Three 4's: $C(10,3)\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^7$

/3 At least one 4: $1-\left(\frac{5}{6}\right)^{10}$

/4 Suppose that someone tells you that at least one die is a 4. What is the probability that you got exactly three 4's?

$= Pr(\text{three 4's} \mid \geq \text{one 4}) = \frac{Pr(\text{three 4's and} \geq \text{one 4})}{Pr(\geq \text{one 4})}$

$= \frac{Pr(\text{three 4's})}{Pr(\geq \text{one 4})} =$ ____

15 points 5. Suppose you select balls from a bin which contains 4 yellow and 6 green balls.

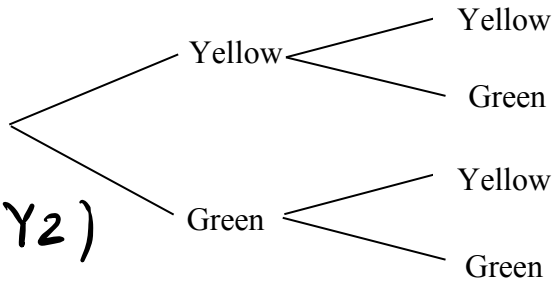
/3 If you select 4 balls, <u>one at a time</u> (without replacement) what is the probability of selecting a <u>green</u> ball and then a <u>yellow</u> ball and then <u>two</u> green balls?

$\frac{6}{10} \cdot \frac{4}{9} \cdot \frac{5}{8} \cdot \frac{4}{7}$

/4 If you select 4 balls, <u>all at the same time</u> (i.e. not one at a time), what is the probability of selecting three yellow balls and one green ball (and order doesn't matter)?

$\frac{C(4,3) \cdot C(6,1)}{C(10,4)}$

/4 Now, if you select 2 balls, first one and then the other, but you don't see the color of the first ball, what is the probability that the second ball is <u>yellow</u>? (Show all pertinent work. Don't just guess/give an answer.) **DO simplify this answer.**



$Pr(Y2) = Pr(Y1 \text{ and } Y2) + Pr(G1 \text{ and } Y2)$

$= \frac{4}{10} \cdot \frac{3}{9} + \frac{6}{10} \cdot \frac{4}{9} = \frac{36}{90} = \frac{4}{10}$

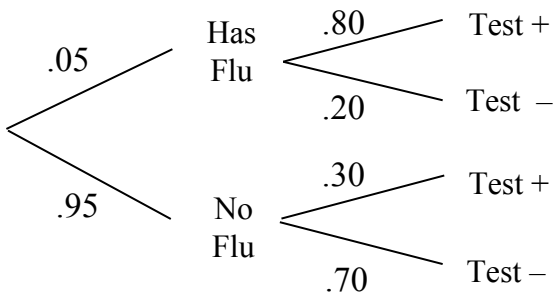
/4 If you select 2 balls, first one and then the other, if the second ball is yellow, what is the probability that the first ball is green? (Show all pertinent work.) **DO simplify this answer.**

$Pr(G1 \mid Y2) = \frac{Pr(G1 \text{ and } Y2)}{Pr(Y2)} = \frac{\frac{6}{10} \cdot \frac{4}{9}}{\frac{36}{90}} = \frac{24}{36} = \frac{6}{9}$

You **DO** need to simplify your answers on this page.

26 points 6. Suppose for a certain group of people, 5% of them have the Flu, and suppose that a certain test which is used to determine whether a person has the Flu gives false negatives 20% of the time and false positives only 10% of the time.

Simplify your answers and fill in the following table. Be sure to **show all pertinent work below**.



For example, $\Pr(Flu \mid - \ Test)$

		Results of test		
		No Test	Positive	Negative
/8	Pr(Has Flu)	.05	.1231	.0148
/8	Pr(Doesn't have Flu)	.95	.8769	.9852

$$\Pr(F|+) = \frac{\Pr(F \text{ and } +)}{\Pr(+)} = \frac{(.05)(.80)}{(.05)(.80)+(.95)(.30)} = \frac{.04}{.325} = .1231$$

$$\Pr(F'|+) = \frac{(.95)(.30)}{} = \frac{.285}{.325} = .8769$$

$$\Pr(F|-) = \frac{(.05)(.20)}{(.05)(.20)+(.95)(.70)} = \frac{.01}{.675} = .0148$$

$$\Pr(F'|-) = \frac{(.95)(.70)}{} = \frac{.665}{.675} = .9852$$

/5 What is the probability that a randomly selected person who has tested positive once would test negative if he/she were tested again?

$$\Pr(-|+) = \frac{\Pr(- \text{ and } +)}{\Pr(+)} = \frac{(.05)(.80)(.20)+(.95)(.30)(.70)}{(.05)(.80)+(.95)(.30)}$$

$$= \frac{.2075}{.3250} = .6385$$

/5 What is the probability that a randomly selected person who tests positive once and negative once actually has the Flu?

$$\Pr(F|+ \text{ and } -) = \frac{\Pr(F \text{ and } + \text{ and } -)}{\Pr(+ \text{ and } -)} = \frac{(.05)(.80)(.20)}{.3250} = .0386$$

You **DO** need to simplify your answers on this page.

8 points 7. Experience shows that younger drivers are more likely to have accidents, and similarly for the very elderly. Consider the information at right for a car insurance company.

Age group	Fraction of all policy holders	Fraction of group in accidents A each year
16 – 24	.20	.10
25 – 64	.60	.03
65 –	.20	.05

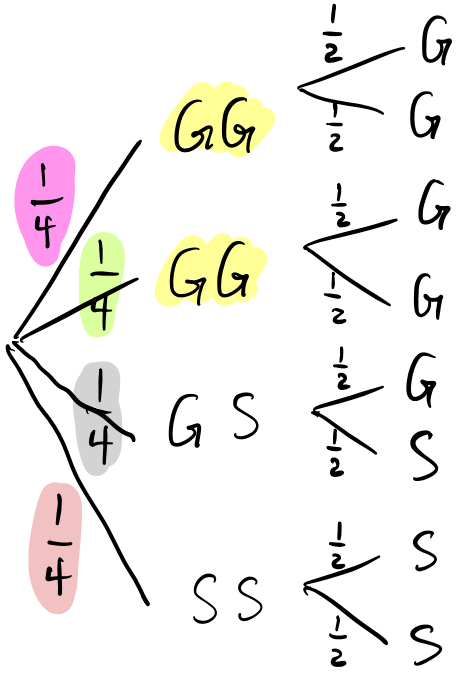
/3 What fraction of their policy holders are in a car accident each year. That is, what is Pr(A), where A is the event of being in an accident?

$$\Pr(A) = (.20)(.10) + (.60)(.03) + (.20)(.05)$$
$$= .048$$

/5 If a policy holder calls in and says they were in an accident, what is the probability they are part of the 25 – 64 age group?

$$\Pr(25\text{-}64 \mid A) = \frac{\Pr(25\text{-}64 \text{ and } A)}{\Pr(A)} = \frac{(.60)(.03)}{.048}$$
$$= .3750$$

5 points 8. Consider four boxes. Two of the boxes contain two gold coins each, one box contains a gold coin and one silver coin, and one box contains two silver coins. Suppose that you select a box at random and then select a coin at random from that box. If the coin is gold, what is the probability that the other coin in the box is gold?



$$\Pr(G/G) = \frac{\Pr(\text{Both } G)}{\Pr(\text{First is } G)}$$
$$= \frac{\frac{2}{4}}{\frac{1}{4}(1) + \frac{1}{4}(1) + \frac{1}{4}\left(\frac{1}{2}\right) + \frac{1}{4}(0)}$$
$$= \frac{\frac{2}{4}}{\frac{5}{8}} = \frac{4}{5}$$