Math 141 Spring 2024 **Midterm Exam 1** **January 26, 2024**

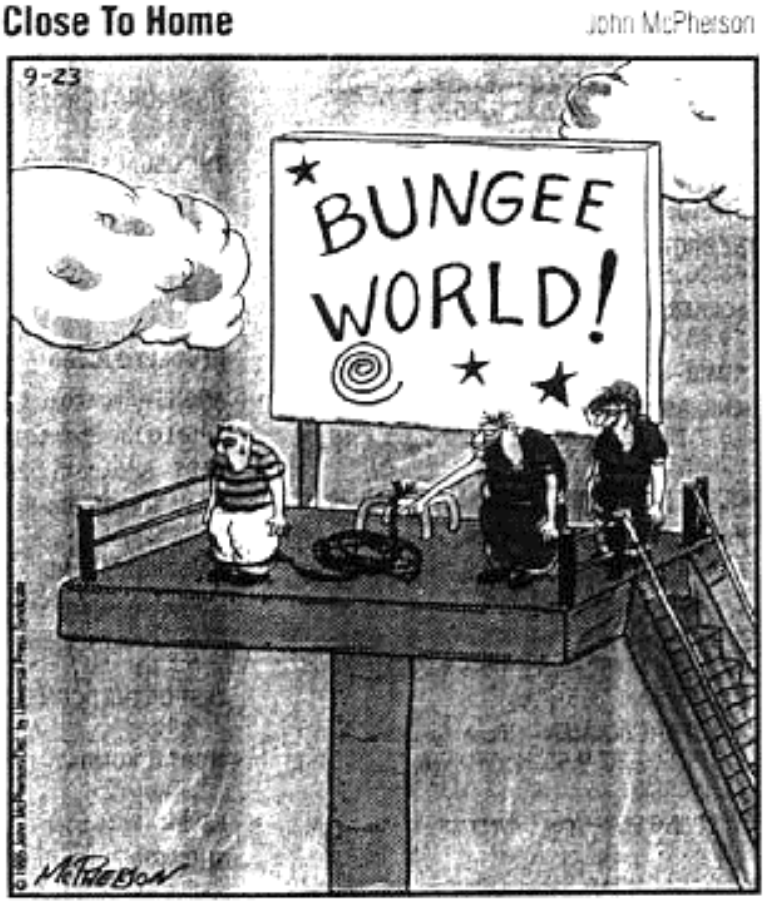
Name: _____

Problem	1	2 / 3	4	5 / 6	7	Total
Possible	20	23	15	24	18	100
Received						

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 x 5 card (both sides) of notes, but no calculator.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.



"Okee-doke! Let's just double-check. We're 130 feet up and we've got 45 yards of bungee cord, that's uh ... 90 feet. Allow for 30 feet of stretching, that gives us a total of ...120 feet. Perfect!"

20 points **1.** Answer each of the following questions. No explanation is needed.

T **F** A system of 3 equations and 3 unknowns could possibly have no solution.

T **F** A system of 3 equations and 3 unknowns could possibly have a unique solution.

T **F** A system of 3 equations and 3 unknowns could possibly have infinite solutions.

T **F** A system of 2 equations and 4 unknowns could possibly have no solution.

T **F** A system of 2 equations and 4 unknowns could possibly have a unique solution.

T **F** A system of 2 equations and 4 unknowns could possibly have infinite solutions.

T **F** Matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is its own inverse.

T **F** $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 21 \\ 12 & 32 \end{bmatrix}$.

T **F** $\begin{matrix} 2x + 3y = a \\ 4x + 5y = b \end{matrix}$ might or might not have a solution, depending on the values of a and b.

T **F** It is possible to choose values for a and b so that $\begin{bmatrix} 2 & 7 \\ a & b \end{bmatrix}$ has an inverse.

14 points **2.** Suppose I have some nickels (5 cents each) and dimes (10 cents each). I have 13 coins total, 90 cents total, and I have 3 more nickels than dimes (so $n = d + 3$). How many of each type of coin do I have? Solve this by coming up with the three equations that correspond to these three conditions (13 coins total, 90 cents total, and 3 more nickels than dimes), then doing Gauss-Jordan Elimination to find the solution(s) to this system of equations. Don't just guess the solution. Or show that there is no solution, if that is the case.

9 points **3.** Rework the previous problem, but with the modified restriction that we have 100 cents, rather than 90 cents (but with the other conditions remaining the same).

15 points **4.** Solve for x, y and z in

$$\begin{aligned} x + y + z &= 1 \\ 2x + y - z &= 0 \\ x + y + 2z &= 1 \end{aligned}$$

by finding the inverse of the coefficient matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & 2 \end{bmatrix}$$

and using it to find the values of x, y and z. Use the Gauss-Jordan Method for finding the inverse. You should not encounter any fractions in finding it. Show work. Don't just guess answers.

10 points **5.** We are interested in solving the following system of equations,

$$3x + 2y = 7$$
$$6x + ay = b$$

where a and b are some constants whose values have not yet been decided. Give an example of values of a and b that result in the system having:

No solution: $a =$ $b =$

One solution: $a =$ $b =$

Infinite solutions: $a =$ $b =$

14 points **6.** Find the solution(s) to each of the following linear systems. If a system has more than one solution, give the general solution and then give *at least two* specific solutions. If a system has no solution, state that.

$$2x + 4y = 7$$
$$-x - y = -2$$

$$x + y - z + 2w = 5$$
$$-x - y + 3z = 7$$

18 points **7.** A company produces two items, but uses up some of each product in the production process, as described by the input-output (consumption) matrix

$$A = \begin{bmatrix} .6 & .2 \\ 0 & .5 \end{bmatrix}.$$

Note for this problem that $(.5)(.4) = .2$, and that $\frac{.5}{.2} = \frac{5}{2}$ and $\frac{.4}{.2} = 2$.

2 points How much of each product would be *consumed* if you *produced* 10 units of each product?

2 points How much of each product is *remaining* if you *produced* 10 units of each product?

2 points How much *more* of each product would be *consumed* if you produced *one more unit* of product 1?

12 points How much would you need to produce in order to *end up* with 10 units of each product? (Use the formula for finding the 2×2 matrix in this problem.) What is one thing about your solution that makes you think it is reasonable, i.e. that it could be the correct answer?