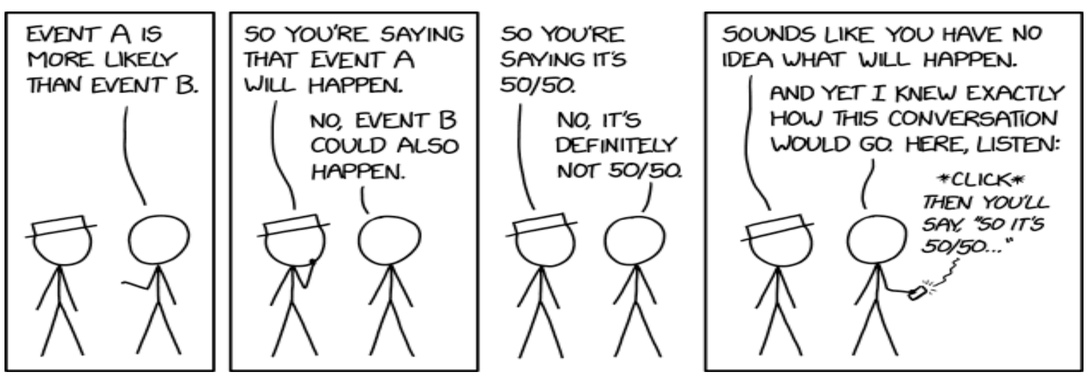
Math 141 Spring 2023 **Midterm Exam 4** **April 3, 2023**

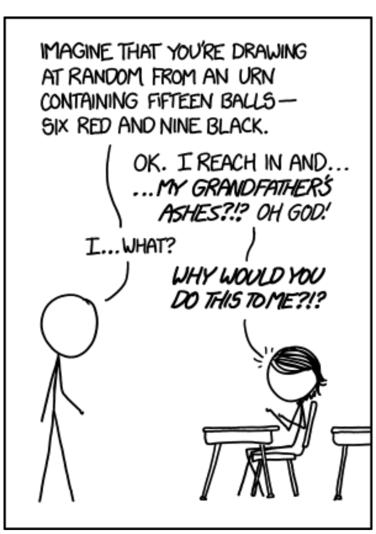
Name: Solutions

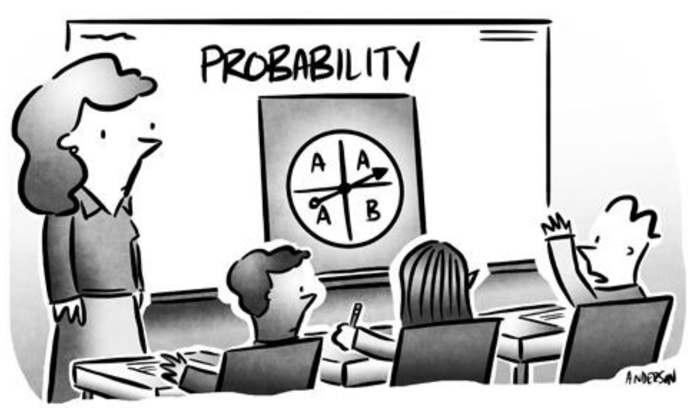
Problem	1	2	3 / 4	5 / 6	7 / 8	Total
Possible	23	15	13	25	24	100
Received						

SIMPLIFY (TO A SINGLE NUMBER) ALL ANSWERS EXCEPT WHERE NOTED.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.

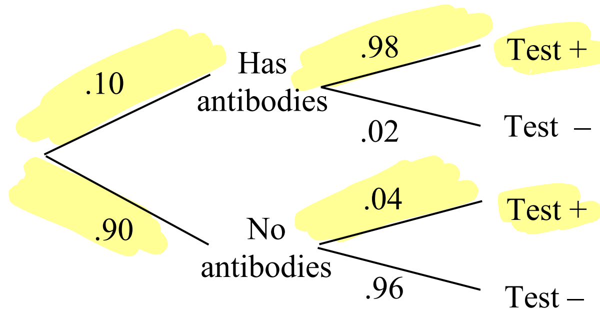






"I know mathematically that A is more likely, but I gotta say, I feel like B wants it more."

23 points 1. Suppose that approximately 10% of Americans have developed antibodies for COVID-19. Suppose that a certain test which is used to determine whether a person has antibodies gives false negatives 2% of the time and false positives 4% of the time.



/13 (a) Fill in the following table. **Be sure to <u>show all</u> <u>pertinent work</u> below the table.**

		Results of test	
	No Test	Positive	Negative
Probability person <u>has</u> antibodies	.10	.731	.002
Probability person <u>does not</u> have antibodies	.90	.269	.998

$$Pr(AB \mid +) = \frac{Pr(AB \text{ and } +)}{Pr(+)} = \frac{(.10)(.98)}{(.10)(.98)+(.90)(.04)} = \frac{.098}{.134} \approx .731$$

$$Pr(AB' \mid +) = \frac{(.90)(.98)}{(.10)(.98)+(.90)(.04)} = \frac{.036}{.134} \approx .269$$

$$Pr(AB \mid -) = \frac{(.10)(.02)}{(.10)(.02)+(.90)(.96)} = \frac{.002}{.866} \approx .002$$

$$Pr(AB' \mid -) = \frac{(.90)(.96)}{(.10)(.02)+(.90)(.96)} = \frac{.864}{.866} \approx .998$$

/5 (b) What is the probability that a person who has tested positive once would test positive if he/she were tested again?

$$Pr(++ \mid +) = \frac{Pr(+ \text{ twice})}{Pr(+ \text{ once})} = \frac{(.10)(.98)^2 + (.90)(.04)^2}{(.10)(.98) + (.90)(.04)} \approx .727$$

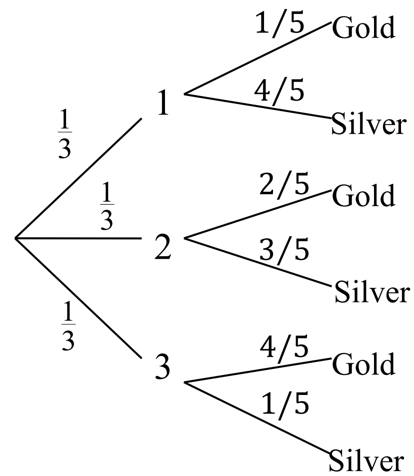
/5 (c) What is the probability that a person who tests positive once and negative once actually has the antibodies?

$$Pr(AB \mid + \text{ and } -) = \frac{Pr(AB \text{ and } + \text{ and } -)}{Pr(+ \text{ and } -)} \approx \frac{(.10)(.98)(.02)}{(.10)(.98)(.02) + (.90)(.04)(.96)}$$

$$\approx .054$$

15 points **2**. Each of three boxes contains five coins:

- Box 1 contains:
  One gold coins.
  Four silver coins.
- Box 2 contains:
  Two gold coins.
  Three silver coins.
- Box 3 contains:
  Four gold coins.
  One silver coin.



A box is randomly selected and one coin is randomly selected from that box.

/2 (a) What is the probability the coin is Silver from Box 2?

$$\left(\frac{1}{3}\right)\left(\frac{3}{5}\right) = \frac{3}{15}$$

/3 (b) What is the probability the coin is Silver from any box?

$$\left(\frac{1}{3}\right)\left(\frac{4}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{5}\right) = \frac{8}{15}$$

/4 (c) If the coin is Silver, what is the probability the box it came from Box 2?

$$Pr\{Box\ 2 / S\} = \frac{Pr(Box\ and\ S)}{Pr(S)} = \frac{\frac{3}{15}}{\frac{8}{15}} = \frac{3}{8}$$

/6 (d) Suppose the first coin is Silver (which we do <u>not</u> put back into the box). If we then choose a second coin from the <u>same</u> box, what is the probability that the other coin we choose is <u>Gold</u>?

in same box

$$Pr\{G/S\} = \frac{Pr(S\ then\ G)}{Pr(S)} = \frac{\left(\frac{1}{3}\right)\left(\frac{4}{5}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{5}\right)\left(\frac{2}{4}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{5}\right)\left(\frac{4}{4}\right)}{\frac{8}{15}}$$

After choosing a coin, only 4 left.

8 points 3. Suppose you roll two dice, and you are interested in their sum. The possible outcomes are listed at right.

Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

/2 (a) What is the probability of rolling an odd sum of ~~8~~ 9 or higher?

$\frac{6}{36}$

/2 (b) Suppose someone can see the dice (you cannot), and tells you the sum is ~~8~~ 9 or higher. What is the probability you rolled an odd number?

$\frac{6}{10}$

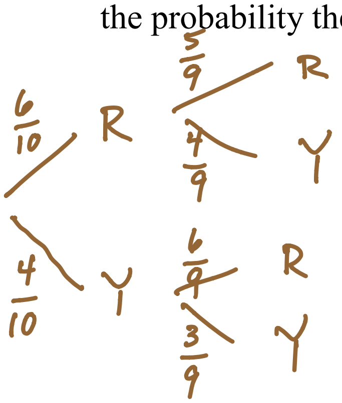
/2 (c) Suppose someone can see the dice (you cannot), and tells you the sum is ~~8~~ 9 or higher. What is the probability one of the dice is a 5?

$\frac{5}{10}$ ($\frac{4}{10}$ if you thought meant exactly one)

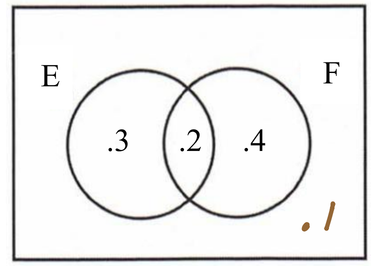
/2 (d) Suppose you can see that one die is a 5, but you cannot see the other die. What is the probability the sum is ~~8~~ 9 or higher?

$\frac{3}{6}$

5 points 4. There are 10 balls: 6 red, 4 yellow. Suppose you select one ball (and you do not put it back). And then you select another. Without seeing the color of the first ball, what is the probability the second ball is yellow? **Show all pertinent details of work**.



$$Pr(Y2) = \frac{6}{10}\cdot\frac{4}{9} + \frac{4}{10}\cdot\frac{3}{9} = \frac{36}{90} = \frac{4}{10}.$$



17 points 5. Suppose $\Pr(E) = .5$, $\Pr(F) = .6$ and $\Pr(E \cap F) = .2$.

Find each of the following.

/1 (a) $\Pr(E') = 1 - .5 = .5$

/1 (b) $\Pr(F') = 1 - .6 = .4$

/2 (c) $\Pr(E \cup F') = .3 + .2 + .1 = .6$ (or $= \Pr(E) + \Pr(F') - \Pr(E \cap F')$ $= .5 + .4 - .3$)

/3 (d) $\Pr(E'|F') = \dfrac{\Pr(E' \cap F')}{\Pr(F')} = \dfrac{.1}{.4}$

/3 (e) $\Pr(E|F') = \dfrac{.3}{.4}$

/3 (f) $\Pr(F|E) = \dfrac{.2}{.5}$

/2 (g) Are events E and F mutually exclusive? Why or why not?

No. Both can occur simultaneously, i.e. $\Pr(E \cap F) > 0$.

/2 (h) Are events E and F independent? Explain, show work.

No. For example, $\Pr(F|E) \neq \Pr(E)$.

8 points 6. We are interested in what proportion of voters in each political party actually vote.

Political party	Proportion registered	Proportion voter turnout (call this "V")
Democrat (D)	.30	.20
Republican (R)	.50	.60
Independent (I)	.20	.70

/1 (a) Suppose we compute that the overall voting rate (i.e. the probability that a randomly selected person votes) $\Pr(V)$ to be .10. How could you tell this is incorrect? (Don't actually compute $\Pr(V)$.)

$\Pr(V)$ should be between .20 and .70.

/5 (b) If a person did not vote, how likely is it that he/she is Democrat? That is, what is $\Pr(D|V')$?

$$\Pr(D|V') = \frac{\Pr(D \text{ and } V')}{\Pr(V')} = \frac{(.30)(.80)}{(.30)(.80) + (.50)(.40) + (.20)(.30)} = .48$$

/2 (c) Are political preference (D, R or I) and Voter turnout independent? Explain, show work.

No. For example, $\Pr(D|V') \neq \Pr(D)$.

YOU DO NOT NEED TO SIMPLY THE ANSWERS FOR THE PROBLEMS ON THIS PAGE.

10 points 7. 5 balls are chosen at random without replacement (that is, without putting the ball back into the basket once it has been chosen). There are 15 balls: 8 green and 7 blue.

/3 (a) What is the probability that all five of the balls you choose are blue?

$$\frac{C(7,5)}{C(15,5)} \quad \text{or} \quad \frac{7}{15}\cdot\frac{6}{14}\cdot\frac{5}{13}\cdot\frac{4}{12}\cdot\frac{3}{11}$$

/3 (b) What is the probability that at least one of them is green?

$$1 - \frac{C(7,5)}{C(15,5)}$$

/4 (c) What is the probability that two of the balls are green and three are blue?

$$\frac{C(8,2)\cdot C(7,3)}{C(15,5)}$$

14 points 8. Suppose that each of 5 persons randomly chooses a letter from the 26-letter alphabet.

/3 (a) What is the probability that all five persons choose the same letter?

$$\frac{26\cdot 1\cdot 1\cdot 1\cdot 1}{26\cdot 26\cdot 26\cdot 26\cdot 26}$$

First person can choose any letter.

/4 (b) What is the probability that two or more of them choose the same letter?

1 − Prob. all different

$$= 1 - \frac{26\cdot 25\cdot 24\cdot 23\cdot 22}{26\cdot 26\cdot 26\cdot 26\cdot 26}$$

/3 (c) What is the probability that all five persons choose a vowel (not necessarily different)?

$$\frac{5\cdot 5\cdot 5\cdot 5\cdot 5}{26\cdot 26\cdot 26\cdot 26\cdot 26}$$

/4 (d) What is the probability that all five persons choose a different vowel (so no repetition)?

$$\frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{26\cdot 26\cdot 26\cdot 26\cdot 26}$$