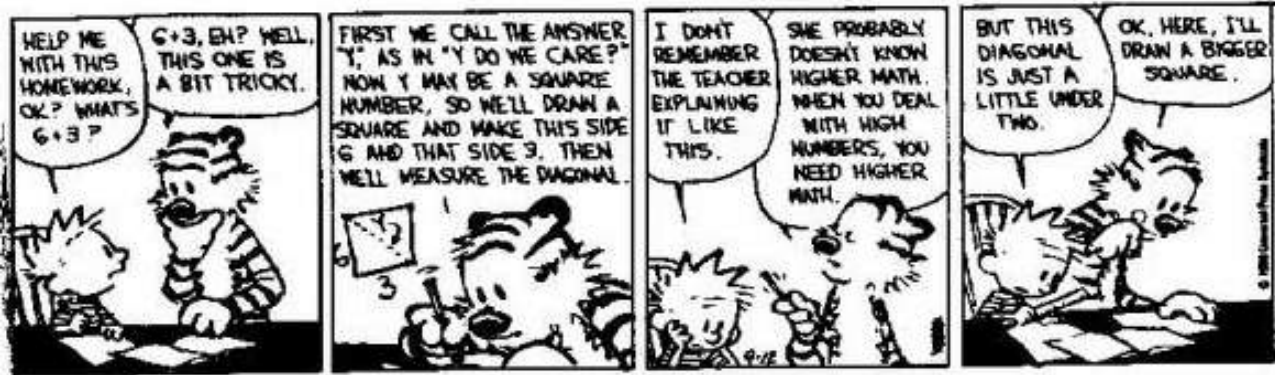


Name: Solutions

Problem	Total
Possible	100
Received	

Don't open exam until told to do so.

**FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.**  
*You will not use a calculator on this exam.*  
**In answering the following questions, not simplify the answers.**  
**For example, leave your answer in the form**  
 $P(5,3)$  or  $12!$  or  $C(4,3) \cdot C(7,4)$  or  $2^5 - 2^3$  or  $7 \cdot 6 \cdot 5$  or ...



- /3 How many different Zoom Meeting IDs are possible if the IDs have 10 digits, each digit is from 0 to 9, and the first digit cannot be a 0.

$$9 \cdot 10^9$$

- /3 How many different passwords can there be if passwords are 8 characters (of any type) from your keyboard? Assume that there are 100 possible characters on the keyboard, and you are allowed to repeat characters in your password.

$$100^8$$

- /3 How many ways are there to divide a group of 24 people up into 4 groups of 6 each?

$$\binom{24}{6, 6, 6, 6} / 4!$$

- /5 Two 11-member soccer teams play a match. After the match, each member of both teams shakes hands with each member of his own team and each member of the other team. How many handshakes take place?

$$11 \cdot 11 + \frac{C(11, 2) \cdot 2}{2}$$

- /2 How many possible outcomes are there if you flip a coin 10 times?

$$2^{10}$$

- /2 How many 5-digit numbers from 10000 to 99999 are there (note that for this problem, 0 cannot be the first digit)?

$$9 \cdot 10^4$$

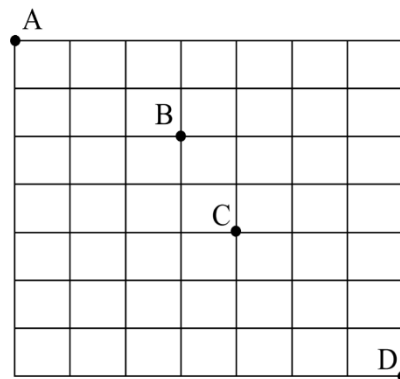
- /5 How many 5-digit numbers 10000 to 99999 (note that for this problem, 0 cannot be the first digit) are there in which three digits are one number and the other two digits are another (e.g. 52522 or 25252 or 52522 or ...)?

$$C(5, 2) \cdot 9 \cdot 9$$

In the diagram at right, if you can only move right (east) and down (south):

/2 How many ways are there to go from A to B?

$$C(5, 3)$$



/5 How many ways are there to go from A to D if you must pass through both B and C?

$$C(5, 3) \cdot C(3, 1) \cdot C(6, 3)$$

A to B          B to C          C to D

/3 If I have 5 shirts, 3 pairs of pants, and 2 pairs of sandals, how many ways can I select an outfit (a shirt, pair of pants and sandals)?

$$5 \cdot 3 \cdot 2$$

/3 How many possible ways are there to award gold, silver, and bronze medals in a race with 20 participants?

$$20 \cdot 19 \cdot 18 = P(20, 3)$$

$$= C(20, 3) \cdot 3!$$

/3 In how many ways can a committee of 10 persons be chosen from 14 married couples if the committee must consist of 4 men and 6 women?

$$C(14, 4) \cdot C(14, 6)$$

/4 In how many ways can a committee of 10 persons be chosen from 14 married couples if a husband and wife cannot both serve on the committee?

$$C(14, 10) \cdot 2^{10} \text{ OR } \frac{28 \cdot 26 \cdot \dots \cdot 10}{10!}$$

- /5 In how many ways could we divide 24 persons into five groups if:  
4 people get an apple, 4 people get a banana, 4 people get a carrot

The other 12 people are simply divided into two groups of 6 people each

$$\binom{24}{4,4,4,6,6} / 2!$$

- /3 How many ways can I choose 2 persons from a group of 20? SIMPLIFY THIS ANSWER TO AN ACTUAL SINGLE NUMBER.

$$C(20, 2) = \frac{20 \cdot 19}{2 \cdot 1} = 10 \cdot 19 = 190$$

- /4 There are ten people. One is Bob. In how many ways can I select a president, vice-president and secretary if Bob must be one of the three selected?

$$1 \cdot 9 \cdot 8 + 9 \cdot 1 \cdot 8 + 9 \cdot 8 \cdot 1$$

Bob Pres. OR VP OR Sec.

- /2 In how many ways can I arrange 15 books on a bookshelf?

$$15!$$

- /5 An ice cream shop offers 10 flavors. In how many ways could you order three scoops?

$$10 + \underbrace{10 \cdot 9}_{C(10,2) \cdot 2} + C(10,3)$$

All same All diff.

- /3 Four artists each have five paintings. In how many ways can the paintings be arranged if there are no restrictions about whose paintings are next to anyone else's?

$$20!$$

- /5 Four artist each have five paintings. In how many ways can the paintings be arranged if each all of the paintings of each artist must be kept side-by-side?

$$4! (5!)^4$$

Order the artists Order each of the four artists' paintings

For the next four questions, there are 20 balls: 12 blue and 8 green. We'll select 5 balls.

/4 How many different samples are possible?

$$C(20, 5)$$

/4 How many samples contain all green balls?

$$C(12, 0) \cdot C(8, 5) = C(8, 5)$$

/4 How many samples contain 2 blue & 3 green balls?

$$C(12, 2) \cdot C(8, 3)$$

/4 How many samples contain 1 or more blue balls?

$$C(20, 5) - C(8, 5)$$

OR  $C(12, 1) \cdot C(8, 4) + \dots + C(12, 5) \cdot C(8, 0)$

For the next five questions, suppose there is a deck of 80 cards of 4 different colors of cards numbered 1 to 20. We will choose 6 cards.

/5 In how many ways can you choose the 6 cards and have three of one number, two of another and one of another (for example, 2 2 2 5 5 9)?

$$\frac{20 \cdot 19 \cdot 18}{C(20, 3) \cdot 3!} \cdot C(4, 3) \cdot C(4, 2) \cdot C(4, 1)$$

/5 In how many ways can you choose 6 cards and have exactly 4 of them be 7's. There could be repetition/duplication in the other 2 cards, but no other 7's (for example, 7 7 7 7 5 5 or 7 7 7 7 5 9 or ...).

$$C(4, 4) \cdot C(76, 2)$$

All four 7's      Two other cards

/4 In how many ways could you have all 6 cards be the same color (a flush)?

$$4 \cdot C(20, 6)$$

Color      Six cards of that color

Extra credit:

/4 In how many ways can you have two straights, each length 6, that do not overlap?

For example: 1 2 3 4 5 6 and 7 8 9 10 11 12

Or: 3 4 5 6 7 8 and 12 13 14 15 16 17

Next page.

Choose the straights

1 to 6 and 7 to 12  
or 8 to 13  
or ...  
or 15 to 20 } 9

2 to 7 and 8 to 13  
or 9 to 14  
or ...  
or 15 to 20 } 8

⋮

9 to 14 and 15 to 20 } 1

So  $1 + \dots + 9 = 45$  in all.  $\downarrow$  4 colours

Then choose the actual card (the color)  
for each of the twelve cards :  $4^{12}$ .

So in total :  $45 \cdot 4^{12}$ .