Math 141 Spring 2023 **Midterm Exam 2** **February 20, 2023**

Name: _____

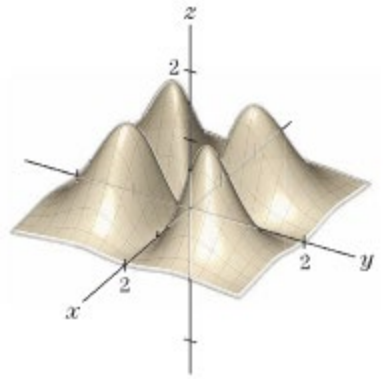
Problem	1	2 / 3	4 / 5	6 / 7	8	9	Total
Possible	8	16	14	22	20	20	100
Received							

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

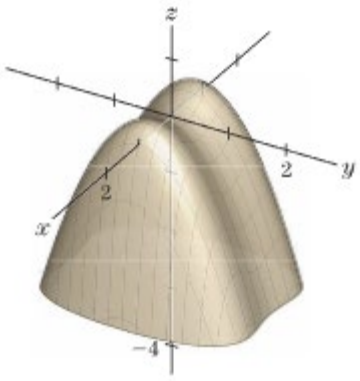
You may use a 3 × 5 card of handwritten notes and a calculator.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.

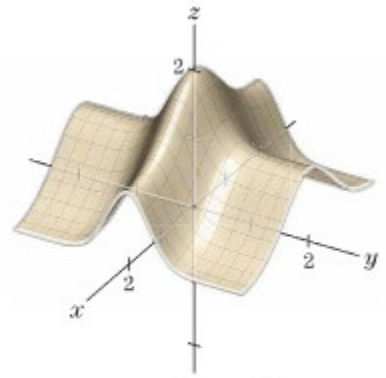
8 points 1. Match the graphs of the following four functions to the level curves show below the functions. Just write a letter (**a** or **b** or **c** or **d**) next to each.



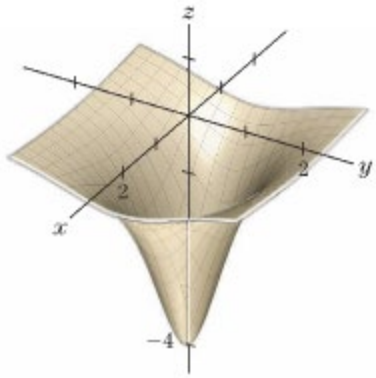
$$z = \frac{15x^2y^2e^{-x^2-y^2}}{x^2+y^2}$$



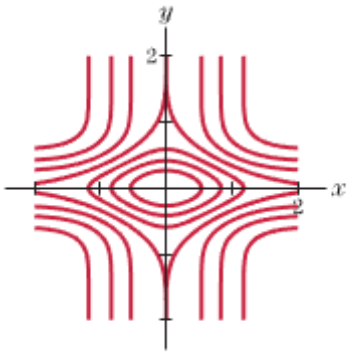
$$z = x^2 - x^4 - y^2$$



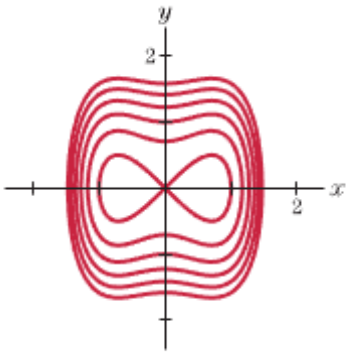
$$z = e^{-x^2} + e^{-4y^2}$$



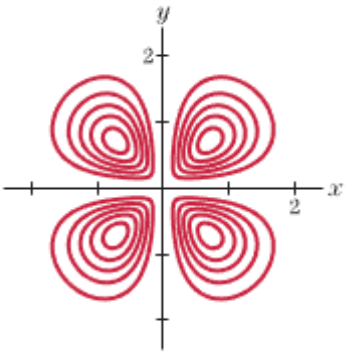
$$z = \frac{-4}{1+x^2+2y^2}$$



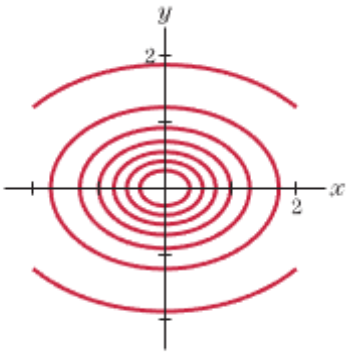
(a)



(b)



(c)



(d)

12 points 2. Consider the production function $f(x,y) = 32x^{1/4}y^{3/4}$, which gives the number of units of goods produced when x units of labor and y units of capital are used.

/4 Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

$\frac{\partial f}{\partial x} =$

$\frac{\partial f}{\partial y} =$

/4 Evaluate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at $x = 81$ and $y = 16$. Note that $81^{1/4} = 3$ and $16^{1/4} = 2$.

$\frac{\partial f}{\partial x}(81,16) =$

$\frac{\partial f}{\partial y}(81,16) =$

/2 Find the marginal productivity of capital of f at $x = 81$ and $y = 16$.

/2 Using above results, approximately what is $f(81,17) - f(81,16)$?

4 points 3. Suppose that $f(10,10) = 100$, $\frac{\partial f}{\partial x}(10,10) = 4$ and $\frac{\partial f}{\partial y}(10,10) = 3$.
Estimate $f(\,11.5\,,9\,)$. Show pertinent details/work.

4 points 4. Suppose the distance D that a car can travel depends on the amount of gas g in the car and the total weight w of the passengers in the car. Circle > 0 or $= 0$ or < 0 for the following derivatives of D.

/2 Should $\frac{\partial D}{\partial g}$ be > 0 or $= 0$ or < 0?

/2 Should $\frac{\partial D}{\partial w}$ be > 0 or $= 0$ or < 0?

10 points 5. Find the point(s) at which $f(x, y) = 5x^2 - 2xy + 2y^2 - 6y + 7$ has minimum(s) and maximum(s), and determine what type of point (min or max or neither) each point is.

14 points 6. Find the following derivatives.

For $f(x,y) = e^{xy^2}$

/3 $\frac{\partial f}{\partial x} =$

/4 $\frac{\partial^2 f}{\partial y \partial x} =$

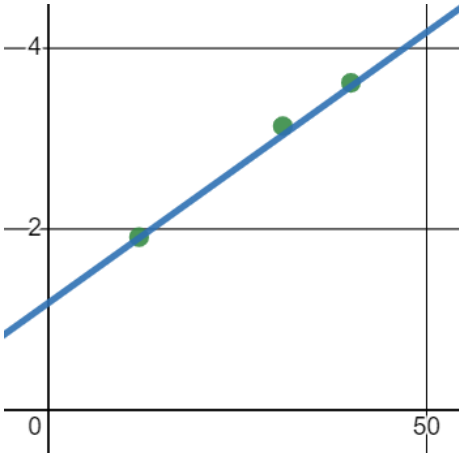
For $g(x,y) = x^2 \cdot \ln(y^3)$

/3 $\frac{\partial g}{\partial y} =$

/4 $\frac{\partial^2 g}{\partial y^2} =$

8 points 7. Suppose that I asked three students how many hours they study per week and what their current GPA is, and found a least squares line based on their responses of

$$GPA \approx 1.18 + .06 * hours\ studied$$



/4 What do the values of 1.18 and .06 tell us?

/2 What GPA would result from studying 10 hours per week?

/2 How many hours per week would you need to study to get a GPA of 3.58?

20 points 8. Suppose we want a very tiny home with dimensions x, y and z to have volume 1 cubic yard, so $xyz = 1$. Suppose that the daily loss (through the walls, ceiling and floor) of heat is given by

$$H = xy + 2xz + 4yz.$$

Find the dimensions of the home which minimize heat loss H.

For this problem, find the solution by substituting $z = \frac{1}{xy}$ into $H = xy + 2xz + 4yz$ and (1) find the values of x and y at which H is minimized, and (2) show that at these x and y values function H is minimized (use second derivatives and $D(x, y)$).

20 points 9. Same as previous problem: minimize

$$H = xy + 2xz + 4yz$$

with the constraint that $xyz = 1$. But now solve this problem by using the Lagrange Multiplier Method.