Math 141 Spring 2023 **Midterm Exam 1** **January 30, 2023**

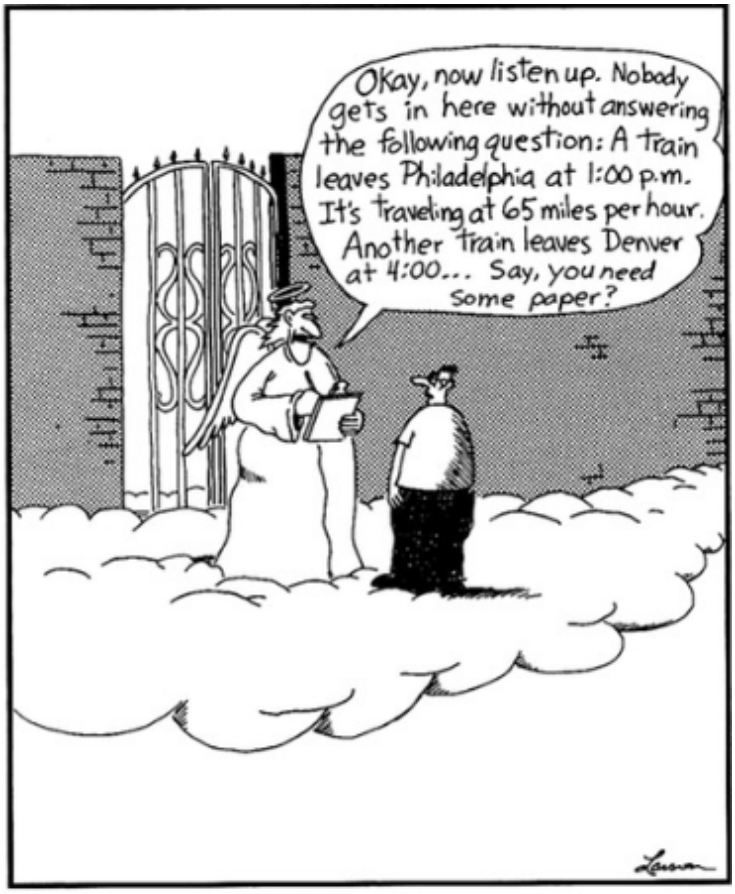
Name: Solutions

Problem	1 / 2	3	4 / 5	6 / 7 / 8	9	Total
Possible	16	20	20	30	14	100
Received						

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 x 5 card (both sides) of handwritten notes and a calculator.

FOR FULL CREDIT, SHOW YOUR WORK FOR TO FINDING EACH SOLUTION.



Math phobic's nightmare

10 points **1.** A college English department purchased two types of tablets for their students to use. Type One costs \$50 each and Type Two costs \$60 each. They purchased of total of 35 tablets for a total of \$2000. How many of each tablet type did they purchase? Solve the system of two equations and two unknowns using either Gauss-Jordan Elimination or using matrices.

x = # Type I
y = # Type II

$50x + 60y = 1400$
$x + y = 35$

$$\begin{bmatrix} 1 & 1 & 35 \\ 50 & 60 & 2000 \end{bmatrix} \longrightarrow \cdots \longrightarrow \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 25 \end{bmatrix}$$

So $x = 10$
$y = 25$

6 points **2.** A dietician wishes to plan a meal around two foods.

Each ounce of food I contains 10% of the daily requirements for carbohydrates, 20% for protein, and 30% for vitamin C.

Each ounce of food II contains 30% of the daily requirements for carbohydrates, 40% for protein, and 50% for vitamin C.

x = Food 1
y = Food 2

Set up (but DO NOT SOLVE) three equations with two unknowns (the two food types) that correspond to this information.

Carbs. $.10x + .30y = 1.00$
Protein $.20x + .40y = 1.00$
Vit. C $.30x + .50y = 1.00$

How many solutions to this problem do you expect? Why?

None : # equations > # unknowns

20 points **3.** Find all of the solutions to the following systems of equations. If there is more than one solution, give the general solution plus two specific solutions. If there is no solution, show that this is the case.

/10
$$\begin{aligned} x - y + z &= 3 \\ -2x + 4y - 4z &= 1 \\ x + y - z &= 4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ -2 & 4 & -4 & 1 \\ 1 & 1 & -1 & 4 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 2 & -2 & 7 \\ 0 & 2 & -2 & 1 \end{array}\right]$$

Can already see there is no solution

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 2 & -2 & 7 \\ 0 & 0 & 0 & -6 \end{array}\right]$$

No solution.

/10
$$\begin{aligned} x + 5y + 3z &= 1 \\ 2x + 9y + 7z &= 4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 1 \\ 2 & 9 & 7 & 4 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 5 & 3 & 1 \\ 0 & -1 & 1 & 2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 8 & 11 \\ 0 & 1 & -1 & -2 \end{array}\right]$$

$x = 11 - 8z$

$y = -2 + z$

z = free

Examples:

x	11	3	-5
y	-2,	-1,	0, etc.
z	0	1	2

10 points **4.** For what value(s) of k will this system of equations have a solution? What is the solution?

$$\begin{aligned} 2x + 4y &= 2 \\ x + 7y &= -4 \\ kx + 8y &= -2 \end{aligned}$$

$$\left[\begin{array}{cc|c} 2 & 4 & 2 \\ 1 & 7 & -4 \\ k & 8 & -2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 5 & -5 \\ 0 & 8-2k & -2-k \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 8-2k & -2-k \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -3k+6 \end{array}\right]$$

Need $-3k+6=0 \Rightarrow k=2$.

and $x=3$, $y=-1$.

Or solve for x & y in

$$\begin{aligned} 2x+4y &= 2 \\ x+7y &= -4 \end{aligned} \Rightarrow \begin{aligned} x &= 3 \\ y &= -1 \end{aligned}$$

Then we need:

$k(3) + 8(-1) = -2$

$\Rightarrow k = 2$.

10 points **5.** Use a matrix equation and a matrix inverse to solve the system of equations

$$\begin{aligned} .2x + .4y &= 1 \\ .8x + .6y &= 2 \end{aligned}$$

$$\begin{bmatrix} .2 & .4 \\ .8 & .6 \end{bmatrix}^{-1} = \frac{1}{(.2)(.6) - (.8)(.4)} \begin{bmatrix} .6 & -.4 \\ -.8 & .2 \end{bmatrix}$$

$(.2)(.6) - (.8)(.4) = -.2$

$$= \begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix}$$

so $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(kind of weird, eh?)

3 points **6.** What is the inverse of $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$?

$$\left[\begin{array}{ccc|ccc} a & 0 & 0 & 1 & 0 & 0 \\ 0 & b & 0 & 0 & 1 & 0 \\ 0 & 0 & c & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a} & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{b} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{c} \end{array}\right]$$

12 points **7.** Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ using the Gauss-Jordan Method $[\,A \mid I\,] \to [\,I \mid A^{-1}]$.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -2 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

15 points **8.** Suppose you have several nickels (5 cents) and dimes (10 cents). How many nickels and dimes would you need so that:

You have 15 coins total.

You have 100 cents (one dollar) total.

You have twice as many nickels as dimes (so $n = 2d$).

Don't just guess an answer—show your work. If there is no solution, show that this is the case.

$$\begin{aligned} n + d &= 15 \\ 5n + 10d &= 100 \\ n - 2d &= 0 \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 15 \\ 5 & 10 & 100 \\ 1 & -2 & 0 \end{array}\right] \to \cdots \to \left[\begin{array}{cc|c} 1 & 0 & 10 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{array}\right]$$

So $n = 10$

$d = 5$

14 points **9.** A company produces two items, but uses up some of each product in the production process, as described by the input-output (consumption) matrix

$$A = \begin{bmatrix} .1 & .5 \\ .3 & .5 \end{bmatrix}$$

/4 If you produced 10 units of each product, how much would remain of product two?

$$\begin{bmatrix} .1 & .5 \\ .3 & .5 \end{bmatrix}\begin{bmatrix} 10 \\ 10 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}, \quad \begin{bmatrix} 10 \\ 10 \end{bmatrix} - \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

/8 If you wanted to end up with 300 units of each product, how much would you need to produce of the two products?

$$(I-A)^{-1} = \begin{bmatrix} .9 & -.5 \\ -.3 & .5 \end{bmatrix}^{-1} = \frac{1}{(.9)(.5)-(-.3)(-.5)} \begin{bmatrix} .5 & .5 \\ .3 & .9 \end{bmatrix}$$

$$.45 - .15 = .30$$

$$= \begin{bmatrix} \frac{.5}{.3} & \frac{.5}{.3} \\ \frac{.3}{.3} & \frac{.9}{.3} \end{bmatrix} = \begin{bmatrix} \frac{5}{3} & \frac{5}{3} \\ 1 & 3 \end{bmatrix}.$$

So produce $\begin{bmatrix} \frac{5}{3} & \frac{5}{3} \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 300 \\ 300 \end{bmatrix} = \begin{bmatrix} 1000 \\ 1200 \end{bmatrix}$

/2 If you wanted to end up with 1 more unit of product two (so in all, 300 units of product one and 301 units of product 2), how much more of the two products would you need to produce?

$$\begin{bmatrix} \frac{5}{3} & \frac{5}{3} \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 300 \\ 301 \end{bmatrix} = \begin{bmatrix} 1000 \text{ and } \frac{5}{3} \\ 1200 \text{ and } 3 \end{bmatrix}$$

so produce an additional $\begin{bmatrix} \frac{5}{3} \\ 3 \end{bmatrix}$, which of course is simply column 2 of $(I-A)^{-1}$.