# Math 141 Spring 2021 Exam 3 Solutions

**Low: 42 Median: 81 High: 101**

**If I based your grade entirely on this exam, then approximately:**

| | |
|---|---|
| **90 – 100** | **A** |
| **70 – 90** | **B** |
| **0 – 70** | **C or below** |

1. $C(10,2)\,(.4)^2(.6)^8 \approx .1209$

   $\mu = .4(10) = 4,\ \sigma = \sqrt{10(.4)(.6)} \approx 1.55$

   $\Pr\left\{\frac{1.5-4}{1.55} \le z \le \frac{2.5-4}{1.55}\right\} \approx \Pr\{-1.60 \le z \le -.95\} = A(-.95) - A(-1.60)$

   $= .1711 - .0548 = .1163$

   $1 - \Pr\{X < 2\} = 1 - [C(10,0)\,(.4)^0(.6)^{10} + C(10,1)\,(.4)^1(.6)^9]$

   $= 1 - [.0060 + .0403] = .9537$

   $\Pr\left\{z \ge \frac{1.5-4}{1.55}\right\} \approx \Pr\{z \ge -1.60\} = 1 - .0548 = .9452$

2.

| $k$ | $\Pr\{X = k\}$ |
|---|---|
| 0 | $\frac{C(5,0)\cdot C(3,2)}{C(8,2)} = \frac{3}{28}\left(= \frac{3}{8}\cdot\frac{2}{7}\right)$ |
| 1 | $\frac{C(5,1)\cdot C(3,1)}{C(8,2)} = \frac{15}{28}\left(= \frac{3}{8}\cdot\frac{5}{7} + \frac{5}{8}\cdot\frac{3}{7}\right)$ |
| 2 | $\frac{C(5,2)\cdot C(3,0)}{C(8,2)} = \frac{10}{28}\left(= \frac{5}{8}\cdot\frac{4}{7}\right)$ |

$E[X] = 0\left(\frac{3}{28}\right) + 1\left(\frac{15}{28}\right) + 2\left(\frac{10}{28}\right) = \frac{5}{4}$

$\sigma^2 = \left(0 - \frac{5}{4}\right)\left(\frac{3}{28}\right) + \left(1 - \frac{5}{4}\right)^2\left(\frac{15}{28}\right) + \left(2 - \frac{5}{4}\right)^2\left(\frac{10}{28}\right) = \frac{45}{112} \approx .4018$

3. From the given area, the $z$ −values at $70 - h$ and $70 + h$ are $-2.5$ and $2.5$, so we need $2.5 = \frac{(70+h)-70}{5} = h \Rightarrow h = 12.5$.

4. $\$5(.1) - \$1(.9) = -\$.40$

For two games:

| Outcome | Total \$ won | Probability |
|---|---|---|
| WW | $\$5 + \$5 = \$10$ | (.1)(.1) |
| WL | $\$5 - \$1 = \$4$ | (.1)(.9) |
| LW | $-\$1 + \$5 = \$4$ | (.9)(.1) |
| LL | $-\$1 - \$1 = -\$2$ | (.9)(.9) |

So expected winnings in two games is $\$10(.01)^2 + \cdots - \$2(.9)^2 = -\$.80$, which isn't a surprise since you should win $-\$.40$ (so lose $\$.40$) per game.

So for 100 games you'd expect a total winnings of $-\$40$ (i.e. lose \$40 in 100 games).

Using the original table given for this problem for a single game:
$\sigma^2 = \left(5 - (-.40)\right)^2(.1) + \left(-1 - (-.40)\right)^2(.9) = 3.24 \Rightarrow \sigma = \sqrt{3.24} \approx 1.8$

5. $\Pr\left(z \le \frac{63-66}{3}\right) = A(-1) = .1587$

$A\left(\frac{63-66}{3}\right) - A\left(\frac{60-66}{3}\right) = A(-1) - A(-2). = .1587 - .0228 = .1359.$

$A(1.5) - A(-1.5) = .9332 - .0668 = .8664.$

$A(z) = .96 \Rightarrow z \approx 1.75 \Rightarrow$ the height $= 66 + 1.75(3) = 71.25.$

$A(z) = .04 \Rightarrow z \approx -1.75 \Rightarrow$ the height $= 66 - 1.75(3) = 60.75.$

Exactly 92% of the heights are between the 96$^{th}$ and 4$^{th}$ percentile.

6. The mean is around 55 (full credit for 50 to 60), since the middle is around 60 but the data are skewed a bit left, and the SD is around 15 (full credit for 10 to 20), since about 2/3 of all the data are within one SD of the mean.

7. If for $X$ we have mean $\mu$ and SD $\sigma$, then for $AX + B$ we have mean $A\mu + B$ and SD $A\sigma$.

| | $X$ | $2X$ | $X+3$ | $2X+3$ |
|---|---|---|---|---|
| | $a$<br>$b$<br>$c$ | $2a$<br>$2b$<br>$2c$ | $a+3$<br>$b+3$<br>$c+3$ | $2a+3$<br>$2b+3$<br>$2c+3$ |
| Mean | 10 | 20 | 13 | 23 |
| Standard deviation | 4 | 8 | 4 | 8 |