Math 141 Fall 2024 **Midterm Exam 3** **October 10, 2024**

Name: Solutions

Problem	1 / 2	3	4 / 5	6 / 7	Total
Possible	32	21	16	31	100
Received					

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

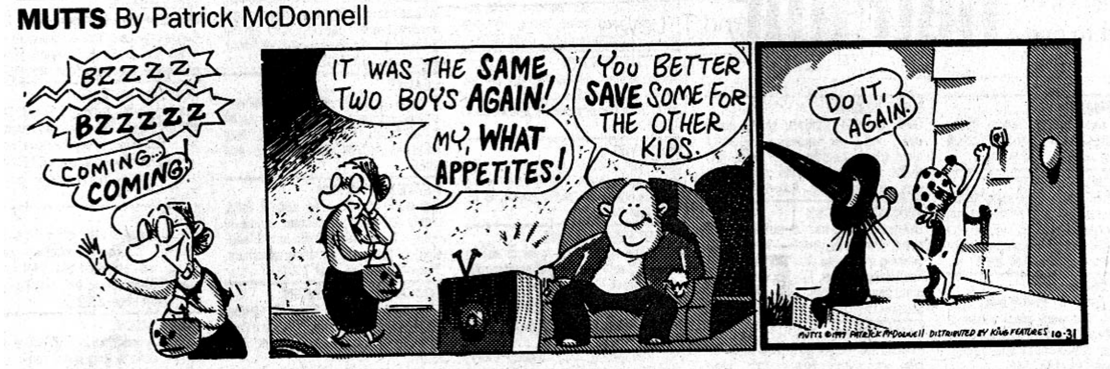
You may use a 3 x 5 card of notes, both sides, and a calculator.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.

FIND THE FINAL ACTUAL VALUE FOR ALL PROBLEMS.

WARPED By Mike Cavna

MUTTS By Patrick McDonnell



17 points 1. Suppose that a person is a 55% free throw shooter. She will shoot 10 shots.

/5 What is the probability of making exactly 7 shots?

$\underbrace{C(10,7)}_{\frac{10\cdot 9\cdot 8}{7\cdot 6\cdot 5}}(.55)^7(.45)^3 = .1665$

/10 What is the probability of making at least 2 of the 10 shots? $= 1 - Pr(\text{not make} \geq 2 \text{ shots})$

$= 1 - Pr(\text{make } 0 \text{ or } 1 \text{ shot})$

$= 1 - [(.45)^{10} + C(10,1)(.45)^1(.55)^9]$

$= 1 - [.000341 + .004162] \approx .9955$

/2 What is the *expected number* of shots she would make? That is, if she were to shoot 10 shots, and then another 10 shots, and then another 10 shots, over and over, then on average how many shots (out of 10) would she make?

$10(.55) = 5.5$

15 points 2. Consider the investment at right.

Return	Probability
−4	.5
2	.1
7	.4

/5 Find the expected value of the investment:

$\mu = (-4)(.5) + 2(.1) + 7(.4) = 1$

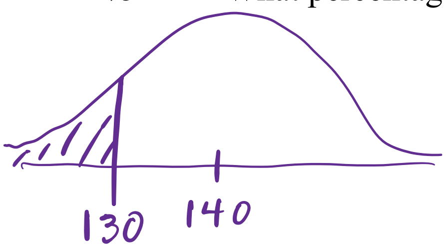
/10 Find the variance and standard deviation of the investment:

$\sigma^2 = (-4-1)^2(.5) + (2-1)^2(.1) + (7-1)^2(.4) = 27$

$\sigma = \sqrt{27} \approx 5.196$

21 points 3. The heights of a certain population of corn plants follow a normal distribution with mean 140 cm and standard deviation 10 cm.

/5 What percentage of the plant heights are 130 cm or less?



$z = \frac{130-140}{10} = -1$

$A(-1) = .1587$

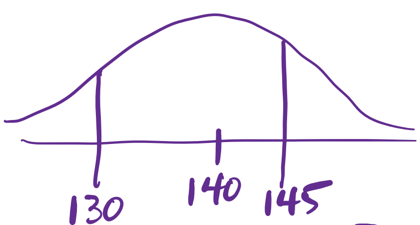
/2 What percentage of the plant heights are 130 cm or more?

$1 - .1587 = .8413$

/1 What percentage of the plant heights are **exactly** 130 cm?

0

/8 What percentage of the plant heights are between 130 and 145 cm?



$A(.5) - A(-1)$

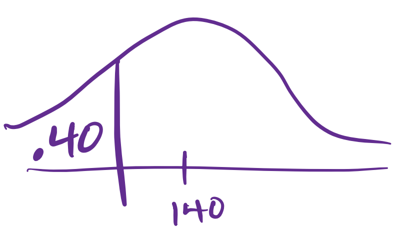
$= .6915 - .1587$

$= .5328$

$z = \frac{145-140}{10}$

$= .5$

/5 What is the plant height at the 40th percentile?



$A(z) = .40 \Rightarrow z \approx -.25$

so height $= 140 - .25(10)$

$= 137.5$

z	$A(z)$	z	$A(z)$
−3.50	.0002	−2.00	.0228
−3.45	.0003	−1.95	.0256
−3.40	.0003	−1.90	.0287
−3.35	.0004	−1.85	.0322
−3.30	.0005	−1.80	.0359
−3.25	.0006	−1.75	.0401
−3.20	.0007	−1.70	.0446
−3.15	.0008	−1.65	.0495
−3.10	.0010	−1.60	.0548
−3.05	.0011	−1.55	.0606
−3.00	.0013	−1.50	.0668
−2.95	.0016	−1.45	.0735
−2.90	.0019	−1.40	.0808
−2.85	.0022	−1.35	.0885
−2.80	.0026	−1.30	.0968
−2.75	.0030	−1.25	.1056
−2.70	.0035	−1.20	.1151
−2.65	.0040	−1.15	.1251
−2.60	.0047	−1.10	.1357
−2.55	.0054	−1.05	.1469
−2.50	.0062	−1.00	.1587
−2.45	.0071	−.95	.1711
−2.40	.0082	−.90	.1841
−2.35	.0094	−.85	.1977
−2.30	.0107	−.80	.2119
−2.25	.0122	−.75	.2266
−2.20	.0139	−.70	.2420
−2.15	.0158	−.65	.2578
−2.10	.0179	−.60	.2743
−2.05	.0202	−.55	.2912

z	$A(z)$	z	$A(z)$
−.50	.3085	1.00	.8413
−.45	.3264	1.05	.8531
−.40	.3446	1.10	.8643
−.35	.3632	1.15	.8749
−.30	.3821	1.20	.8849
−.25	.4013	1.25	.8944
−.20	.4207	1.30	.9032
−.15	.4404	1.35	.9115
−.10	.4602	1.40	.9192
−.05	.4801	1.45	.9265
.00	.5000	1.50	.9332
.05	.5199	1.55	.9394
.10	.5398	1.60	.9452
.15	.5596	1.65	.9505
.20	.5793	1.70	.9554
.25	.5987	1.75	.9599
.30	.6179	1.80	.9641
.35	.6368	1.85	.9678
.40	.6554	1.90	.9713
.45	.6736	1.95	.9744
.50	.6915	2.00	.9772
.55	.7088	2.05	.9798
.60	.7257	2.10	.9821
.65	.7422	2.15	.9842
.70	.7580	2.20	.9861
.75	.7734	2.25	.9878
.80	.7881	2.30	.9893
.85	.8023	2.35	.9906
.90	.8159	2.40	.9918
.95	.8289	2.45	.9929

10 points 4. Suppose you flip 3 coins. We are interested in the number of heads, which we call X.

/8 Find the probability distribution for the number of heads:

k	$\Pr(X = k)$
0	$C(3,0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3 = \frac{1}{8}$
1	$C(3,1)\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 = \frac{3}{8}$
2	$C(3,2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 = \frac{3}{8}$
3	$C(3,3)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0 = \frac{1}{8}$

TTT

HTT THT TTH

HHT HTH THH

HHH

/2 What is the expected number of heads?

$0\left(\frac{1}{8}\right) + 1\left(\frac{3}{8}\right) + 2\left(\frac{3}{8}\right) + 3\left(\frac{1}{8}\right) = \frac{12}{8} = 1.5 \left(= 3 \cdot \underbrace{\frac{1}{2}}_{\text{Prob. of heads}} \right)$

6 points 5. Suppose there are 80 people to be tested for Covid. If each person has his/her own individual test, then 80 tests will be administered and analyzed. Suppose instead that all 80 are tested ("pooled") together. If that one test is negative, then we're done: all 80 people are Covid-free. But if the pooled test is positive, then one (or more) person in the group has Covid, thus all 80 will then be tested individually. Find the expected number of tests to be administered in this "pooled testing" approach. You can start by completing the table below. Assume that 0.5% (so 0.005) of the population being tested actually have Covid. Simplify/evaluate your answer below for expect number of tests.

Outcome of pooled test	Total number of tests given	Probability
Positive	81	$1-(.995)^{80} \approx .3304$
Negative	1	$(.995)^{80} \approx .6696$

Expected number of tests per 80 people =

$81(.3304) + 1(.6696) \approx 27.43$

11 points 6. Find (and **show appropriate work**) the mean, variance and standard deviation of the following ten values:

2 2 2 2 2 5 5 10 10 10

$$\mu = \frac{2\cdot 5 + 5\cdot 2 + 10\cdot 3}{10} = 5$$

$$\sigma^2 = \frac{(2-5)^2\cdot 5 + (5-5)^2\cdot 2 + (10-5)^2\cdot 3}{10 \leftarrow \text{or } 9} = 12$$

$$\sigma = \sqrt{12} \approx 3.46$$

20 points 7. Suppose we roll a die 10 times. We are interested in getting fives. 

/5 What is the probability of getting exactly 2 fives?

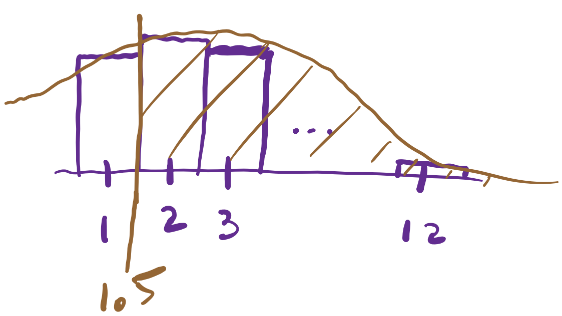
$$\underbrace{C(10,2)}_{\frac{10\cdot 9}{2\cdot 1}}\cdot\left(\tfrac{1}{6}\right)^2\left(\tfrac{5}{6}\right)^{10} = .2907$$

<u>Use the normal curve to approximate the next two probabilities.</u>

/9 What is the probability of getting **at least** 2 fives?

$$\mu = 10\left(\tfrac{1}{6}\right) = \tfrac{5}{3}$$

$$\sigma = \sqrt{10\left(\tfrac{1}{6}\right)\left(\tfrac{5}{6}\right)} \approx 1.1785$$

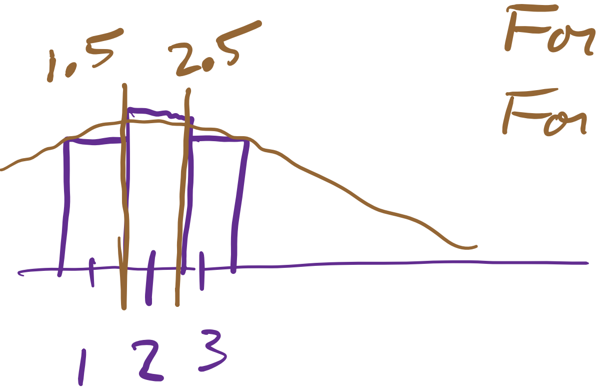


$$z = \frac{1.5 - \frac{5}{3}}{1.1785} \approx -.15$$

$$A(-.15) = .4404$$

$$1 - .4404 = .5596$$

/6 What is the probability of getting exactly 2 fives? (Remember to use the **normal curve** to approximate this value.)



For 1.5, $z \approx -.15$

For 2.5, $z \approx \dfrac{2.5 - \frac{5}{3}}{1.1785} \approx .70$

$$A(.70) - A(-.15)$$
$$= .7580 - .4404 = .3176$$