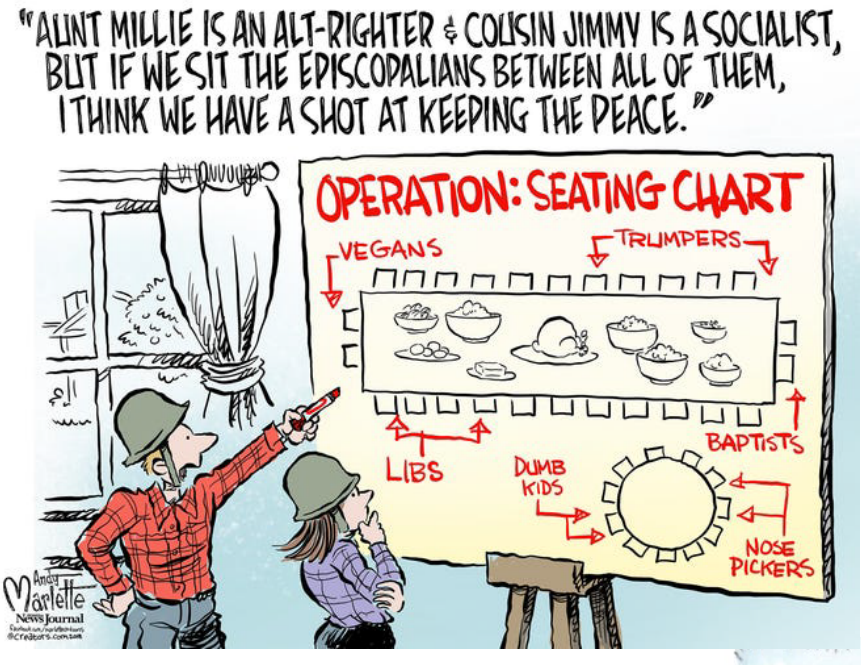
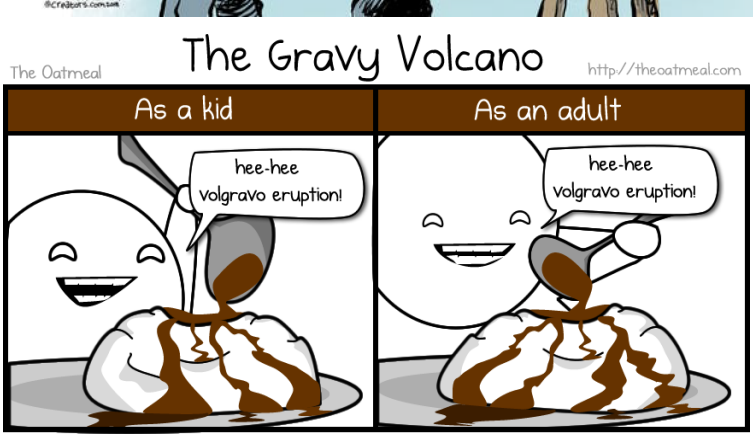
Math 141 Fall 2023 **Midterm Exam 4** **November 9, 2023**

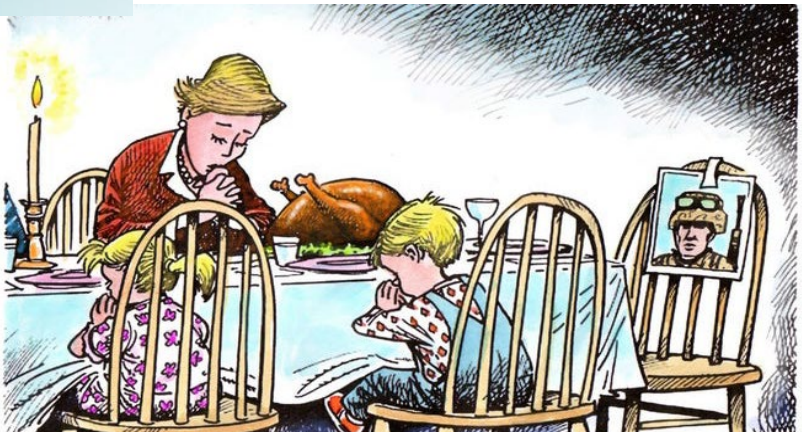
Name: Solutions

Problem	1	2	3 / 4	5 / 6	7 / 8	Total
Possible	23	15	13	25	24	100
Received						

SIMPLIFY (TO A SINGLE NUMBER) ALL ANSWERS, EXCEPT IN PROBLEM 8.
You may use a calculator and a 3 x 5 card of handwritten notes.
FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.

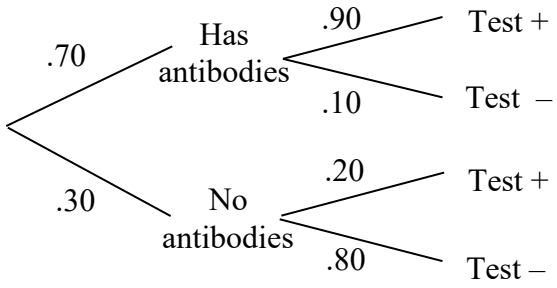






70%

23 points 1. Suppose that approximately [redacted] of Americans have developed antibodies for COVID-19. Suppose that a certain test which is used to determine whether a person has antibodies gives false negatives 10% of the time and false positives 20% of the time.



/13 Fill in the following table. **Be sure to <u>show all pertinent work</u> below the table**.

	Results of test: No Test	Positive	Negative
Probability person <u>has</u> antibodies	.70	.913	.226
Probability person <u>does not</u> have antibodies	.30	.087	.774

$Pr(A|+) = \frac{(.7)(.9)}{(.7)(.9)+(.3)(.2)} = \frac{.63}{.69} = .913$

$Pr(NA|+) = \frac{(.3)(.2)}{(.7)(.9)+(.3)(.2)} = \frac{.06}{.69} = .087$

} Add up to 1.

$Pr(A|-) = \frac{(.7)(.1)}{(.7)(.1)+(.3)(.8)} = \frac{.07}{.31} = .226$

$Pr(NA|-) = \frac{(.3)(.8)}{(.7)(.1)+(.3)(.8)} = \frac{.24}{.31} = .774$

} Add up to 1.

/5 What is the probability that a person who has tested <u>positive once</u> would test <u>positive again</u> if he/she were tested again?

$Pr(\text{++ again} \mid \text{+ once}) = \frac{Pr(\text{++ twice})}{Pr(\text{+ once})} = \frac{(.7)(.9)^2+(.3)(.2)^2}{(.7)(.9)+(.3)(.2)} = \frac{.579}{.690} = .839$

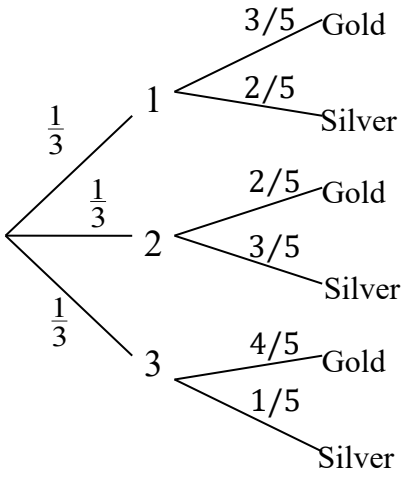
/5 What is the probability that a person who tests <u>positive once and negative once</u> does <u>not</u> have the antibodies?

$Pr(NA \mid + \text{ and } -) = \frac{Pr(NA \text{ and } + \text{ and } -)}{Pr(+ \text{ and } -)} = \frac{(.3)(.2)(.8)}{(.7)(.9)(.1)+(.3)(.2)(.8)}$

$= \frac{.048}{.111} = .432$

15 points 2. Each of three boxes contains five coins:

- Box 1 contains:
  Three gold coins.
  Two silver coins.
- Box 2 contains:
  Two gold coins.
  Three silver coins.
- Box 3 contains:
  Four gold coins.
  One silver coin.



A box is randomly selected and one coin is randomly selected from that box.

/2 What is the probability the coin is Gold from Box 2?

$$\frac{1}{3} \cdot \frac{2}{5} = \frac{2}{15}$$

/3 What is the probability the coin is Gold from any box?

$$\frac{1}{3} \cdot \frac{3}{5} + \frac{1}{3} \cdot \frac{2}{5} + \frac{1}{3} \cdot \frac{4}{5} = \frac{9}{15}$$

/4 If the coin is Gold, what is the probability the box it came from Box 2?

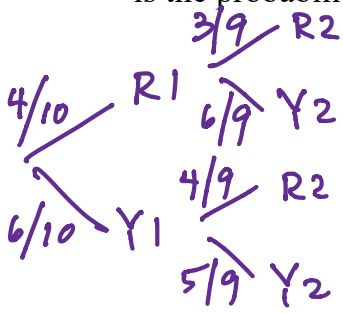
$$Pr(\text{Box } 2 \mid G) = \frac{Pr(\text{Box } 2 \text{ and } G)}{Pr(G)} = \frac{\frac{2}{15}}{\frac{9}{15}} = \frac{2}{9}$$

/6 Suppose the first coin is Gold (which we do not put back into the box). If we then choose a second coin from the same box, what is the probability that the other coin we choose is Silver?

$$Pr(\text{Same box } S2 \mid G1) = \frac{Pr(G1 \text{ and Same box } S2)}{Pr(G1)} = \frac{\frac{1}{3} \cdot \frac{3}{5} \cdot \frac{2}{4} + \frac{1}{3} \cdot \frac{2}{5} \cdot \frac{3}{4} + \frac{1}{3} \cdot \frac{4}{5} \cdot \frac{1}{4}}{\frac{9}{15}}$$

$$= \frac{\frac{16}{60}}{\frac{9}{15}} = \frac{4}{9}.$$

5 points 3. There are 10 balls: 4 red, 6 yellow. Suppose you select one ball (and you do not put it back). And then you select another. Without seeing the color of the first ball, what is the probability the second ball is yellow? **Show all pertinent details of work.**



$$Pr(Y_2) = Pr(R_1 \cap Y_2) + Pr(Y_1 \cap Y_2)$$
$$= \frac{4}{10} \cdot \frac{6}{9} + \frac{6}{10} \cdot \frac{5}{9} = \frac{54}{90} = \frac{6}{10}$$

8 points 4. Suppose you roll two dice, and you are interested in their sum. The possible outcomes are listed at right.

Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

/2 What is the probability of rolling a sum that is both odd and 7 or higher?

$$\frac{6+4+2}{36} = \frac{12}{36}$$

/2 Suppose someone can see the dice (you cannot), and tells you the sum is 7 or higher. What is the probability you rolled an odd sum?

$\frac{12}{21}$ ← odd and ≥ 7

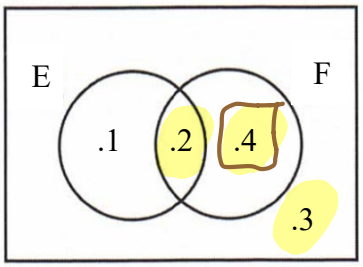
/2 Suppose someone can see the dice (you cannot), and tells you the sum is odd. What is the probability that the sum is 7 or higher?

$\frac{12}{18}$ ← odd and ≥ 7

/2 Suppose you can see that one die is a 3, but you cannot see the other die. What is the probability the sum is 7 or higher?

$\frac{3}{6}$

17 points 5. Suppose $\Pr(E) = .3$, $\Pr(F) = .6$ and $\Pr(E \cap F) = .2$.



Find each of the following.

/1 $\Pr(E') =$ .7

/1 $\Pr(F') =$ .4

/2 $\Pr(E' \cup F) =$ .2 + .4 + .3

/3 $\Pr(F|E) =$ .2 / .3

/3 $\Pr(E'|F) =$ .4 / .6

/3 $\Pr(F|E') =$ .4 / .7

/2 Are events E and F mutually exclusive? Why or why not? No. $\Pr(E \cap F) \neq 0$

/2 Are events E and F independent? Explain, show work. No.

$\Pr(F|E) \neq \Pr(F)$, etc.

8 points 6. A certain corporation has employees in three regions of the world. The proportions of employees in each region who are bilingual (speak ≥ 2 languages) are given at right.

Region	Proportion of all employees	Proportion who are bilingual
North America	.40	.30
Europe	.50	.60
Other	.10	.80

/3 What proportion of all of this corporation's employees are bilingual?

$\Pr(B) = (.4)(.3) + (.5)(.6) + (.1)(.8)$

$= .50$ (so $\Pr(B') = 1 - .50 = .50$)

/5 If a person is not bilingual, how likely is it that he/she is from the North America region?

$$\Pr(NA|B') = \frac{\Pr(NA \text{ and } B')}{\Pr(B')} = \frac{(.4)(.7)}{.5}$$

$$= .56$$

10 points 7. The table below shows the responses when 300 employees were asked if they thought that their company's executives are paid too much.

	Yes	No	Neutral	Total
Male	70	45	35	150
Female	95	10	45	150
Total	165	55	80	300

Find the probability that a person…

/2 Answered yes: $165/300$

/2 Is male: $150/300$

/2 Is male, given that the person answered yes. $70/165$

/2 Answered yes, given that he is male: $70/150$

/2 Is being male independent of saying "Yes" that executives are paid too much? Explain.

No. $Pr(M|yes) \neq Pr(M)$, etc.

14 points 8. Suppose that 4 persons each randomly choose a number between 1 and 10, inclusive.

YOU DO NOT NEED TO SIMPLY THESE FOUR ANSWERS TO A SINGLE NUMBER.

/3 What is the probability that all 4 persons choose the same number?

$$\frac{10 \cdot 1 \cdot 1 \cdot 1}{10 \cdot 10 \cdot 10 \cdot 10}$$

/4 What is the probability that 2 or more of them choose the same number?

$$1 - \frac{10 \cdot 9 \cdot 8 \cdot 7}{10 \cdot 10 \cdot 10 \cdot 10}$$ } Prob. all different

/3 What is the probability that all 4 persons choose an odd number (not necessarily different)?

$$\frac{5 \cdot 5 \cdot 5 \cdot 5}{10 \cdot 10 \cdot 10 \cdot 10}$$

/4 What is the probability that all 4 persons choose a different odd number?

$$\frac{5 \cdot 4 \cdot 3 \cdot 2}{10 \cdot 10 \cdot 10 \cdot 10}$$