Name: _____ Solutions _____

Problem	Total
Possible	100
Received	

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3×5 (both sides) of handwritten notes.
You will <u>not</u> use a calculator for this exam.

In answering the following questions, <u>except on page 4</u>, do not simplify the answers. For example, leave your answer in the form
$P(5,3)$ or $12!$ or $C(4,3) \cdot C(7,4)$ or $2^5 - 2^3$ or $7 \cdot 6 \cdot 5$ or …

CLOSE TO HOME By John McPherson
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Before giving out candy, the Gernsteads
required that trick-or-treaters first watch a
short video on dental care.

Deer Halloweens

/3 In how many ways can I arrange 6 French books and 8 novels on a shelf if the novels all have to be next to teach other?

$$7 \cdot 8! \cdot 6!$$

(where to put the group of novels, relative to the French books.)

F F F F F F

/3 In how many ways can I select 3 books from 8?

$$C(8,3)$$

/3 In how many ways can I arrange 3 books of 8 on a shelf?

$$P(8,3)$$

/3 How many social security numbers are there if the only number that cannot be assigned is 000-00-0000?

$$10^9 - 1$$

/3 In how many ways can I divide a group of 20 people into two groups of 5 and one group of 10?

$$\binom{20}{5,5,10} \Big/ 2!$$

/3 How many ways can you give a $1, $2, and a $5 bill to 3 of your 8 friends (one bill for each friend that you choose)?

$$P(8,3)$$

/3 In how many ways can you choose 3 green balls and 2 red ones from a basket containing 5 green, 4 red and 6 yellow balls?

$$C(5,3) \cdot C(4,2)$$

/3 How many numbers between 100000 and 999999 are there in which three digits are one number and the other three digits are another (e.g. 525252, 225552, 555222, etc.)?

$$\binom{6}{3,3} \Big/ 2! \cdot 9 \cdot 9 \qquad \text{or} \qquad C(5,2) \cdot 9 \cdot 9$$

/3 A certain country has 10-digit phone numbers: 3-digit area code, 7-digit phone number. How many 10-digit phone numbers are there if the only restriction is that 0 cannot be the first digit of either the area code or of the phone number?

$$9 \cdot 10 \cdot 10 \cdot 9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$$

/3 In how many ways could we divide a basketball team of 15 players into the 5 starters and the 10 substitutes?

$$\binom{15}{5,10} = C(15,5)$$

/3 How many 4-digit numbers from 1000 to 9999 are there in which the digits are all different?

$$9 \cdot 9 \cdot 8 \cdot 7$$

/3 How many ways could I choose three different toppings to put on three scoops of ice cream: one scoop of vanilla and two scoops of chocolate.

$$C(10,3) \cdot 3 \leftarrow \text{which topping goes on vanilla}$$
Choose the toppings

/3 If I have 10 pieces of candy, all of different types, and 10 kids show up at my door, in how many different ways could I hand the candy out, one piece to each kid?

$$10!$$

For the next four questions, there are 5 couples: 1 boy and 1 girl per couple; so in all, 5 boys and 5 girls, and 10 persons total.

/3 In how many ways can we select 7 persons consisting of 3 boys and 4 girls?

$$C(5,3) \cdot C(5,4)$$

/3 In how many ways can we select 4 persons if all 4 must come from different couples, i.e. you cannot have a boy and girl from the same couple?

$$C(5,4) \cdot 2^4$$
Choose the couples ↑ Boy or girl from each couple

/3 In how many ways can the 10 persons be seated side-by-side if the boys must be seated next to each other and the girls must to be seated next to each other?

or $\left.\begin{array}{l} B \cdots B \, G \cdots G \\ G \cdots G \, B \cdots B \end{array}\right\}$ $2 \cdot 5! \cdot 5!$ OR $10 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
↑ Anyone Same gender Other gender

/3 In how many ways can the 10 persons be seated side-by-side if each couple must be seated together?

$$5! \cdot 2^5 \qquad \text{OR} \quad 10 \cdot 1 \cdot 8 \cdot 1 \cdots \cdot 2 \cdot 1$$
Order the 5 couples   Order each couple

For the next five questions, there are 5 boys and 5 girls, so 10 persons total.
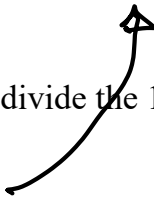
/5   In how many different ways could we choose 1 boy and 1 girl?

$$5 \cdot 5 = 25$$

/5   In how many different ways could we choose any 2 of the 10?

$$C(10,2) = \frac{10!}{2! \, 8!} = \frac{10 \cdot 9}{2 \cdot 1} = 45$$

/5   In how many different ways could we divide the 10 into two groups of sizes 2 and 8?

Same as

/5   In how many different ways could we choose a president and a vice-president from these 10 kids?

$$10 \cdot 9$$

/5   If the girls are named Alice, Barb, Carol, Deb, and Elizabeth, in how many different ways could we seat the 10 kids side-by-side if <u>the girls must all be next to each other and in alphabetical order</u>?
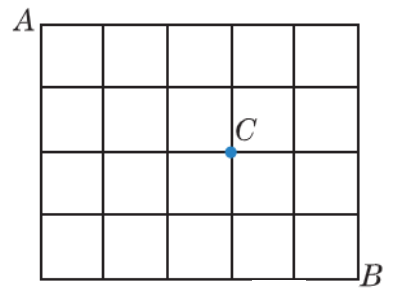
B   B   B   B   B

$$6 \cdot 5! \cdot 1 = 720$$

Where to put girls   Order Boys   one way to put girls in alpha. order

For the next two questions, you will go from point A to point B, and you will either go right (East) or down (South).

A grid diagram with point A at top-left, point C at center, and point B at bottom-right.

/3 How many shortest routes are there from A to B?

$$C(9,5) \quad OR \quad C(9,4)$$

/3 How many shortest routes are there from A to B that pass through C?

$$C(5,3) \cdot C(4,2) \quad OR \quad C(5,2) \cdot C(4,2)$$
$$A \text{ to } C \qquad C \text{ to } B$$

For the next two questions, 20 runners will run a race. We are interested in how all 20 runners finish (rather than just the top 3, for example).

/3 How many different outcomes are there, i.e. how many different orderings are there of how the 20 runners finish the race?

$$20!$$

/3 One of the runners is named Bob. How many different outcomes are there in which Bob finishes first?

$$1 \cdot 19!$$

For the next three questions, suppose there is a deck of 60 cards of 4 different colors of cards numbered 1 to 15. In how many ways can you choose 8 cards so that:

/4 You have a pair: 2 of one number, and the other 6 cards are all __different__ other numbers, for example, 5 5 1 3 4 7 9 15.

$$C(15,7) \cdot C(7,1) \cdot C(4,2) \cdot [C(4,1)]^6$$
The 7 numbers   The double

/4 All of them are the same color, for example, 1 3 4 5 7 9 11 15, all of one color.

$$C(4,1) \cdot C(15,8)$$
The color   The numbers

/4 You have a really full house: 3 of one number, 3 of another number, and 2 of another number, for example, 5 5 5 7 7 11 11 11.

$$C(15,3) \cdot C(3,2) \cdot C(4,3) \cdot C(4,3) \cdot C(4,2)$$
The 3 numbers   Which to   have triples of

5

$\downarrow$ C(4, 2) · C(56, 6)

would be the answer if the question
were to have two 5's and then
any six other cards