Math 141 Fall 2023 **Exam 2** **October 3, 2023**

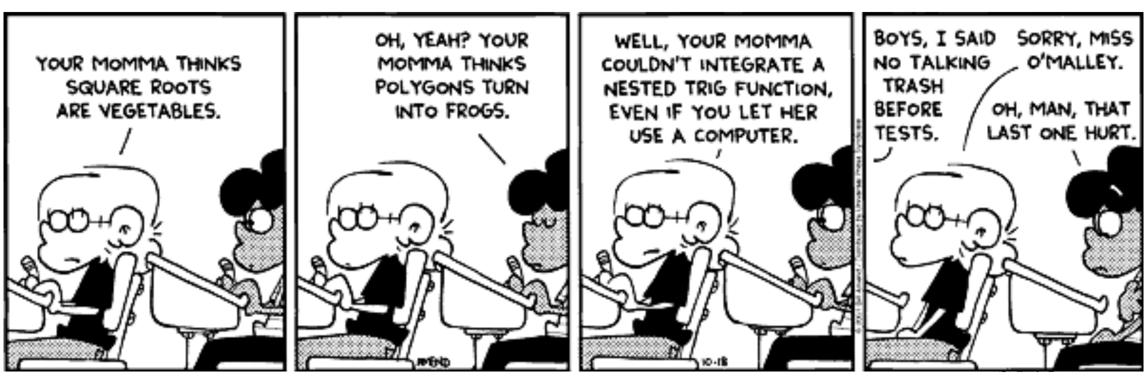
Name: Solutions

Problem	1 / 2	3	4 / 5 / 6	7 / 8	Total
Possible	28	20	32	20	100
Received					

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 × 5 card of handwritten notes and a calculator.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.



"Sit and stay were no problem but she's hit a wall with multivariable calculus."

12 points 1. Find the **three** numbers whose sum is as small as possible and whose product is 1000. **Don't just guess/give the answer. Show all pertinent work. Use the method of Lagrange Multipliers to find your solution.**

$$F(x,y,z,\lambda) = x+y+z+\lambda(1000-xyz)$$

① $\frac{\partial F}{\partial x} = 1-\lambda yz = 0 \Rightarrow \lambda = \frac{1}{yz}$

② $\frac{\partial F}{\partial y} = 1-\lambda xz = 0 \Rightarrow \lambda = \frac{1}{xz}$

③ $\frac{\partial F}{\partial z} = 1-\lambda xy = 0 \Rightarrow \lambda = \frac{1}{xy}$

$\Rightarrow \frac{1}{yz} = \frac{1}{xz} \Rightarrow y = x$

$\Rightarrow \frac{1}{xz} = \frac{1}{xy} \Rightarrow y = z$

In constraint: $xyz = 1000$

$y \cdot y \cdot y = 1000 \Rightarrow y = 10,\ x = 10,\ z = 10.$

16 points 2. Suppose that $f(x,y) = -2x^2 + 4xy - 3y^2 - 12x - 8y$. Find the value(s) of x and y at which f has relative minimums and/or maximums and determine type of each point (min, max, neither, etc.).

$\frac{\partial f}{\partial x} = -4x+4y-12$ $\qquad \frac{\partial^2 f}{\partial x^2} = -4$ $\qquad \frac{\partial^2 f}{\partial y \partial x} = 4$

$\frac{\partial f}{\partial y} = 4x-6y-8$ $\qquad \frac{\partial^2 f}{\partial y^2} = -6$ $\qquad \frac{\partial^2 f}{\partial x \partial y} = 4$

So $D(x,y) = (-4)(-6) - (4)(4) = 8$

$= 0: \quad -4x+4y = 12$

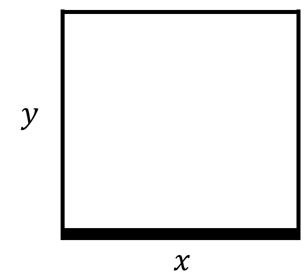
$= 0: \quad 4x-6y = 8$

$\Rightarrow x = -13,\ y = -10$

$D(-13,-10) = 8 > 0$, so max or min.

$\frac{\partial^2 f}{\partial x^2}(-13,-10) = -4$, so max.

20 points 3. We want to build a fence. Three of the sides will be of wood which is \$10/foot and the fourth side will be of stone which is \$30/foot. So the total cost (see the diagram at right) would be $C(x, y) = 40x + 20y$. Find the dimensions x and y which maximize the area enclosed by these fences if we have a total of \$400 to spend. **Show all pertinent work. Use the method of Lagrange Multipliers to find your solution.**



Constraint: $40x + 20y = 400$.

$F(x, y, \lambda) = xy + \lambda(400 - 40x - 20y)$

$\frac{\partial F}{\partial x} = y - 40\lambda = 0 \Rightarrow \lambda = \frac{y}{40}$

$\Rightarrow y = 2x$

$\frac{\partial F}{\partial y} = x - 20\lambda = 0 \Rightarrow \lambda = \frac{x}{20}$

So $40x + 2(2x) = 400$

$x = \frac{400}{80} = 5$

$y = 10.$

9 points 4. Suppose that $f(7,8) = 100$ and $\frac{\partial f}{\partial x}(7,8) = 5$ and $\frac{\partial f}{\partial y}(7,8) = 10$.
Estimate each of the following.

/3 $f(10,8) \approx$ $100 + 3(5) = 115$
since $x \nearrow 3$

/3 $f(7,6) \approx$ $100 - 2(10) = 80$
since $y \searrow 2$

/3 $f(10,6) \approx$ $100 + 3(5) - 2(10) = 95$

15 points 5. Suppose level of production is $f(x,y) = 60x^{1/2}y^{1/2}$ for x units of labor and y units of capital. Find <u>and interpret</u> each of the following:

/3 $f(4,9) =$ $60\sqrt{4}\sqrt{9} = 360$
Production level.

/5 $\frac{\partial f}{\partial x}(4,9) =$ $60\left(\frac{1}{2}\right)(4)^{-\frac{1}{2}}(9)^{\frac{1}{2}} = 45$
If $x \nearrow 1$, then $f \nearrow 45$.

/5 $\frac{\partial f}{\partial y}(4,9) =$ $60(4)^{\frac{1}{2}}\left(\frac{1}{2}\right)(9)^{-\frac{1}{2}} = 20$
If $y \nearrow 1$, then $f \nearrow 20$.

/2 What is the marginal productivity of capital at $(x,y) = (4,9)$.

8 points 6. Suppose demand D for a certain production is a function of its price p, its quality q, the price c of a product in competition with your product, and the amount of advertising money a you spend on marketing the product. Circle one of > 0 or $= 0$ or < 0 for each of the following derivatives of D.

/2 $\frac{\partial D}{dp}$ should be > 0 or $= 0$ or < 0 — If $p \nearrow$, then $D \searrow$

/2 $\frac{\partial D}{dq}$ should be > 0 or $= 0$ or < 0 — $q \nearrow$ $D \nearrow$

/2 $\frac{\partial D}{dc}$ should be > 0 or $= 0$ or < 0 — $c \nearrow$ $D \nearrow$

/2 $\frac{\partial D}{da}$ should be > 0 or $= 0$ or < 0 — $a \nearrow$ $D \nearrow$

10 points 7. Suppose that a least squares line is found using past sales data which relates sales S (in number of units sold per month) to amount of advertising money spent A (in *thousands* of dollars spent per month):

$$S = 100 + 1.5A$$

/4 What do the values of 100 and 1.5 represent? (Don't just say slope and y-intercept—tell me what they mean in this problem.)

If $A \uparrow 1$ (\$1 thousand more advertising per month)
then $S \uparrow 1.5$ (sell 1.5 more units per month)

If $A = 0$ (no adv. \$ spent),
then $S = 100$ (sell 100 units per month)

According to the above model:

/3 How much should be spent on advertising in order to sell 140 units per month?

$140 = 100 + 1.5A$

$\Rightarrow A = \dfrac{140 - 100}{1.5} = \$26\tfrac{2}{3}$ thousand

/3 What number of sales would result from spending \$10 thousand dollars per month on advertising?

$S = 100 + 1.5(10) = 115$ units sold per month.

10 points 8. Find the following:

/4 $\frac{\partial}{\partial x} e^{e^{e^{xy}}} = e^{e^{e^{xy}}} \cdot e^{e^{xy}} \cdot e^{xy} \cdot x$

/3 $\frac{\partial}{\partial x} x^3 y^5 = 3x^2 y^5$

/3 $\frac{\partial^2}{\partial y \partial x} x^3 y^5 = 3x^2 \cdot 5y^4$