Math 141 Fall 2023 **Midterm Exam 1** **September 8, 2023**

Name: ____

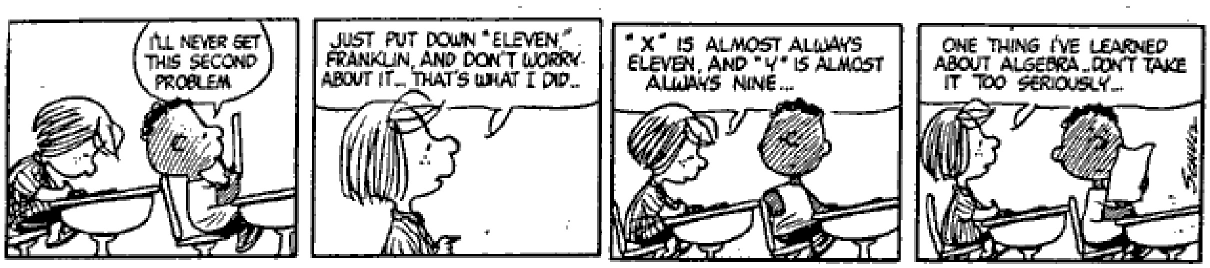
Problem	1	2	3	4	5	6/EC	Total
Possible	10	20	18	23	20	9	100
Received							

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 × 5 card of handwritten notes and a calculator.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.

PEANUTS /Charles Schulz



10 points 1. Answer each of the following True/False questions. No explanation is needed.

T **F** The system of equations $\begin{matrix} 2x - 3y + 5z = 2 \\ 2x - 3y + 4z = a \end{matrix}$ may or may not have an infinite number of solutions, depending on what a is.

T **F** The system of equations $\begin{matrix} 2x - 3y + 5z = 2 \\ 2x - 3y + 5z = a \end{matrix}$ may have exactly one solution, depending on what a is.

T **F** The matrix $\begin{bmatrix} 1 & 2 \\ 3 & a \end{bmatrix}$ has an inverse no matter what a is.

T **F** If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ does not have an inverse, then $AX = B$ will not have a solution.

T **F** For $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, the system $AX = B$ may not have a solution, depending on what B is.

20 points 2. A group will order some burgers, fries and drinks. Each burger costs \$10, each order of fries costs \$6, and each drink costs \$2. Some (or all) of the following conditions will be met:

1. They will order 9 items.
2. They will spend a total of \$50.
3. They will order one more burger than drinks.

In each of the following three problems:

- If there is one solution, find it.
- If there is no solution, state so and explain why (show some matrix work).
- If there are infinite solutions, <u>find the general solution and find two particular solutions</u>.

/10 How many of each item will they buy if they need to meet conditions 1 and 2?

/10 How many of each item will they buy if they need to meet conditions 1, 2 and 3?

18 points 3. Suppose a certain economy consists of two sectors. Suppose that this economy has the input-output (consumption) matrix $A = \begin{bmatrix} .2 & .3 \\ .4 & .2 \end{bmatrix}$.

/2 How much of each product would be *consumed* if you *produced* 100 units of each product?

/2 How much of each product is *remaining* if you *produced* 100 units of each product?

/2 How much *more* of each product would be *consumed* if you produced *one more unit* of product 2?

/10 How much would you need to produce in order to *end up* with 100 units of each product? (Use the formula for finding the 2×2 matrix in this problem.) What is one thing about your solution that makes you think it is reasonable, i.e. that it could be the correct answer?

/2 How much *more* of each product would be needed if you wanted to *end up* with *one more* unit of product 2 (so 100 units of product 1, and 101 units of product 2).

23 points 4. Consider the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$.

/10 Use Gauss-Jordan elimination to find the inverse of A.

/4 Use A^{-1} to find the solution to $\begin{aligned} x + y + z &= 2 \\ x + 2y + 3z &= 2 \\ x + y + 2z &= 3 \end{aligned}$.

/9 Now use Gauss-Jordan elimination to find the solution to $\begin{aligned} x + y + z &= 2 \\ x + 2y + 3z &= 2 \\ x + y + 2z &= 3 \end{aligned}$.

20 points 5. Find the solutions to each of the following linear systems. If a system has more than one solution, give the general solution and then give two specific solutions. If a system has no solution, state that. Show work—don't just write answers.

/8

$$\begin{aligned} x + y - 2z + 3w &= 5 \\ -2x - 2y + 2z - 2w &= 6 \end{aligned}$$

/6

$$\begin{aligned} 2x - 6y &= 10 \\ -4x + 12y &= -20 \end{aligned}$$

/6

$$\begin{aligned} n + d &= 12 \\ n - 5d &= 0 \\ 5n + 10d &= 70 \end{aligned}$$

9 points 6. Each of the following is the final matrix of a Gauss-Jordan elimination process. Give the solutions to the corresponding systems of linear equations. You can use x and y (and z, if needed) for the unknowns.

Final matrix	Solution
$\left[\begin{array}{cc\|c} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right]$	
$\left[\begin{array}{ccc\|c} 1 & -1 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$	
$\left[\begin{array}{cc\|c} 1 & 0 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$	

2 points EC. Extra credit. Be sure to show your work.

If $A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $A^5 = \begin{bmatrix} 3 & 5 \\ 5 & 8 \end{bmatrix}$, what is A?