Managing That Weird Thing Called Light

Light has unique properties which are often nonintuitive. For example, the speed of light is a constant 3.00×10^8 m/s regardless of its wavelength, λ, or frequency, ν.

$$c = \lambda\nu$$

This is not true for other forms of energy, such as sound energy. Also, light behaves simultaneously as a wave or a particle (a photon), a characteristic shared by quantum sized particles but not macroscopic particles or objects.

1. Violet light has a wavelength of about 410 nm. What is the frequency of this light?

$$c = \lambda\nu \quad \text{so} \quad \nu = \frac{c}{\lambda} = \frac{3.00\times10^{8}\,\frac{\text{m}}{\text{s}}}{410\times10^{-9}\,\text{m}} = 7.32\times10^{14}\,\text{s}^{-1} = 7.3\times10^{14}\,\text{s}^{-1}$$

2. It is very difficult to transmit electromagnetic radiation appreciable distances underwater. The U.S. Nave has developed a system for communicating with submerged submarines using very long wavelengths. The system uses radiowaves with a frequency of 76 Hz. What is the wavelength of this radiation? (The Hz is the unit of frequency and is equivalent to /s or s^{-1}. It is an honorific given to Heinrich Hertz[1])

$$c = \lambda\nu \quad \text{so} \quad \lambda = \frac{c}{\nu} = \frac{3.00\times10^{8}\,\frac{\text{m}}{\text{s}}}{76\ \text{Hz}} = 3.95\times10^{6}\ \text{m} = 3{,}900\ \text{km(!)}$$

As postulated by Max Planck[2] and rigorously verified , the energy possessed by a photon is independent of its intensity and dependent upon its frequency.

$$E = h\nu$$

where *E* is the energy of the photon in J, ν is the frequency of oscillating energy in the photon, and *h* is Planck's constant, 6.626×10^{-34} J·s

3. The most prominent line in the line spectrum of aluminum is found at 396.15 nm. What is the energy of one photon of light with this wavelength?

$$E = h\nu \ \text{ and } \ \nu = \frac{c}{\lambda} \quad \text{so} \quad E = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\,\text{J}\cdot\text{s})(2.9997\times10^{8}\,\frac{\text{m}}{\text{s}})}{396.15\times10^{-9}\ \text{m}} = 5.0173\times10^{-19}\,\text{J}$$

[1] Heinrich Rudolf Hertz (1857-1894)
[2] Max Karl Ernst Ludwig Planck (1858-1947)

4. What is the energy of the light in problem 3 in kJ/mol?

$$E = 5.0173\times10^{-19}\,\text{J}(6.022\times10^{23}\,{}^{\big/}_{\text{mol}})(\tfrac{1\text{ kJ}}{1000\text{ J}}) = 302.14\,\tfrac{\text{kJ}}{\text{mol}}$$

5. The most prominent line in the spectrum of magnesium is 285.2 nm. Other prominent lines are found at 383.8 and 518.4 nm. In what region of the electromagnetic radiation spectrum is each line? Which line is the most energetic?

The 285.2 nm line is in the ultraviolet and is the most energetic; the other two lines are in the visible light.

6. Referring to problem 5, would the most energetic line have sufficient energy to initiate a chemical reaction by breaking a chemical bond which has a bond strength of 405 kJ/mol?

$$E = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\,\text{J}\cdot\text{s})(2.9997\times10^{8}\,\tfrac{\text{m}}{\text{s}})}{285.2\times10^{-9}\text{ m}}\times6.022\times10^{23}\,{}^{\big/}_{\text{mol}}\times\tfrac{1\text{ kJ}}{1000\text{ J}} = 419.7\,\tfrac{\text{kJ}}{\text{mol}}$$

This photon is sufficient in energy to initiate this chemical reaction.

7. The amount of power being produced by the Sun is 1370 W/m^2. Assume for this problem that the Sun produces only 525 nm light. What is the approximate number of photons striking 1 m^2 of illuminated Earth surface per second? (A watt, W, is a J/s)

$$P = 1370\,\tfrac{\text{W}}{\text{m}^2} = 1370\,\frac{\text{J}/\text{s}}{\text{m}^2}$$

$$E = \frac{(6.626\times10^{-34}\,\text{J}\cdot\text{s})(3.00\times10^{8}\,\tfrac{\text{m}}{\text{s}})}{525\times10^{-9}\text{ m}} = 3.786\times10^{-19}\,\text{J}(/_{\text{photon}})$$

$$N = \frac{1370\,\frac{\text{J}/\text{s}}{\text{m}^2}}{3.786\times10^{-19}\,\text{J}/_{\text{photon}}} = 3.62\times10^{21}\,\frac{\text{photons}/\text{s}}{\text{m}^2}$$

Albert Einstein[3] earned the 1921 Nobel Prize in physics for the discovery of the law of the photoelectric effect – the concept which definitively validates the particle nature of light.

$$KE_{e^-} = h\nu - \Phi_b$$

8. A particular metal has a binding energy of 6.7 × 10^{-19} J/atom. What will be the kinetic energy of the photoelectrons produced if the metal is illuminated by x-ray radiation with a wavelength of 1.1 nm?

$$\nu = \frac{c}{\lambda} = \frac{3.00\times10^{8}\ \frac{m}{s}}{1.1\times10^{-9}\,m} = 2.73\times10^{17}\,Hz$$

$$KE_{e^-} = (6.626\times10^{-34}\,J\cdot s)(2.73\times10^{17}\,Hz) - 6.7\times10^{-19}\,J = 1.800\times10^{-16}\,J$$

9. To cause a metallic cesium surface to eject an electron, a photon with a minimum energy of 200(±10) kJ/mol is required. What is the longest possible wavelength of light that will cause cesium metal to lose an electron?

$$E_{photon} = 200\,\frac{kJ}{mol}\times\frac{1000\ J}{kJ}\times\frac{1\ mol}{6.022\times10^{23}\ photons} = 3.21\times10^{-19}\,J$$

$$\lambda = \frac{hc}{E} = \frac{(6.626\times10^{-34}\,J\cdot s)(3.00\times10^{8}\ \frac{m}{s})}{3.21\times10^{-19}\,J}\times10^{9}\ \frac{nm}{m} = 599\ nm\ (600\pm10\ nm)$$

The concept of the quantized nature of matter and, in particular, the quantized electronic energy levels in atoms was first postulated by Niels Bohr[4] in 1913. Through the Rydberg[5] equation (1890) it is possible to calculate all of the line emission energies for hydrogen atom.

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

[3] Albert Einstein (1879-1955)
[4] Niels Henrik David Bohr (1885-1962)
[5] Johannes Robert Rydberg (1854-1919)

R_H (the Rydberg constant) in this form of the Rydberg equation has the value of 1.0974×10^7 m^{-1}. It is possible to convert the Rydberg constant to units of joules and rewrite the equation

$$\Delta E = -R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

where, in this case, the Rydberg constant is 2.179×10^{-18} J. This new form of the Rydberg equation predicts with a high level of accuracy the differences in energy between energy levels in the hydrogen atom. The negative sign corrects the equation for direction of energy travel; *i.e.*, if an electron falls from high energy to low, the atom gives off light (an "exothermic" process). When $n_f = 2$, the equation is known as the Balmer equation since if predicts the wavelengths or energies of the Balmer emission lines (visible light lines) of hydrogen atom almost exactly.

10. What is energy of the photon produced when an electron in an excited-state hydrogen atom falls from $n = 4$ to $n = 2$?

$$\Delta E = -R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = -2.179 \times 10^{-18}\,\text{J} \left(\frac{1}{2^2} - \frac{1}{4^2} \right) = -4.086 \times 10^{-19}\,\text{J}$$

The sign just indicates that the atom lost electronic energy

11. What is the wavelength of the photon in problem 10? What color is this photon?

$E = -4.086 \times 10^{-19}\,\text{J}$

for wavelength calculation, the sign can be ignored.

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\,\text{J} \cdot \text{s})(3.00 \times 10^8\,\frac{\text{m}}{\text{s}})}{4.086 \times 10^{-19}\,\text{J}} \times 10^9\,\frac{\text{nm}}{\text{m}} = 486.5\text{ nm}$$

This photon is green.

12. The Rydberg equation can be used to calculate the energy (or wavelength) of a photon absorbed by a hydrogen atom. Could the photon produced in problem 10 be absorbed by a ground-state hydrogen atom?

No. There will be no (can be no) energy difference for the Lyman series that corresponds to that photon energy. For proof, use the Rydberg equation to show that *n*=2 to *n*=1 is already higher in energy than *n*=4 to *n*=2, thus no other transition ending in *n*=1 can possibly have the same energy.

But why should the energy levels (or in Bohr's model, the electron orbits) be quantized at all? When quantum-sized particles in motion, they also possess wave-like characteristics. This was first postulated by Louis de Broglie[6] in 1925 to help explain quantization in the Bohr model of the hydrogen atom. De Broglie proposed that, if the electron is behaving as a wave (like a photon), then the electron "wave" must constructively interfere with itself as it completes an orbit (inspect the drawing of a standing wave on an orbit, below). If the wave does not constructively interfere (that is, create a standing wave), the electron will move in toward or away from the nucleus until the distance traveled in orbit establishes a standing wave. Despite Bohr's atomic theory having been invalidated, de Broglie's equation that relates the apparent wavelength of a moving particle to its mass and velocity has been verified in a wide variety of experimental situations.

$$\lambda = \frac{h}{mv}$$

The reader is referred to a discussion of de Broglie waves, Compton scattering, Young's double-slit experiment, the photoelectric effect for a more in-depth treatment of wave-particle duality.

The presentation of quantum mechanics by Schrödinger[7] in 1926 revolutionized physics and, ultimately chemistry. The details of quantum mechanics are complex but not inaccessible and are a work of exquisite beauty and "...*spring from true genius*..." (Einstein,1926).

13. What is the apparent wavelength of an electron (9.11×10^{-31} kg) moving at 90.0% the speed of light?

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\,\mathrm{J\cdot s}}{(9.11\times10^{-31}\,\mathrm{kg})(0.90\cdot3.00\times10^{8}\,\tfrac{\mathrm{m}}{\mathrm{s}})} = 2.70\times10^{-12}\,\mathrm{m}$$

$$\lambda = 2.70\times10^{-3}\,\mathrm{nm}\ \text{(equivalent to } \gamma\text{-ray wavelengths)}$$

Almost everything we know about the microscopic nature of matter is encoded in the way light interacts with the matter. For example, quantum mechanics is inextricably tied to the energies of light absorption and emission from atoms and molecules. The energy of light absorbed is the difference in energy between the atomic or molecular orbitals in the substance.

14. An atom has an emission line at 630.7 nm. What is the difference in energy between the orbitals which produce this emission line?

$$\Delta E_{\mathrm{orbitals}} = E_{\mathrm{photon}} = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\,\mathrm{J\cdot s})(3.00\times10^{8}\,\tfrac{\mathrm{m}}{\mathrm{s}})}{630.7\times10^{-9}\,\mathrm{nm}} = 3.152\times10^{-19}\,\mathrm{J}$$

[6] Louis Victor Pierre Raymond duc de Broglie (1892-1987)
[7] Erwin Rudolf Josef Alexander Schrödinger (1887-1961)

15. A dye molecule has a wavelength of maximum absorption of 495 nm. What is the difference in energy between the molecular orbitals which account for this absorption?

$$\Delta E_{\text{orbitals}} = E_{\text{photon}} = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\,\text{J}\cdot\text{s})(3.00\times10^{8}\,\tfrac{\text{m}}{\text{s}})}{495\times10^{-9}\,\text{nm}} = 4.02\times10^{-19}\,\text{J}$$