Integrative Problem of Hess' Law and Heat Evolution

Problem 7-85

A particular natural gas consists, in mole percents, of 83.0%CH_4, 11.2%C2H6, and 5.8% C3H8. A 385-L sample of this gas, measured at 22.6°C and 739 mmHg, is burned at constant pressure in an excess of oxygen gas. How much heat, in kilojoules, is evolved in the combustion reaction?

This multi-step problem requires calculation of:

1) $\Delta H_{\text{combustion}}$ for each reaction
2) total quantity of gas (in moles)
3) the quantity, in moles, of each gas
4) the quantity of heat produced by each gas and summing the individual heats together

1) There's no table of $\Delta H_{\text{combustion}}$ in the textbook so use Hess' law to calculate the $\Delta H_{\text{combustion}}$ of each fuel gas from $\Delta H_{\text{formation}}$ (Table 7.2).

$CH_4(g) + 2\ O_2(g) \rightarrow CO_2(g) + 2\ H_2O(g)$

$$\Delta H^{CH_4}_{\text{combustion}} = (-393.5\text{ kJ} + \left(2\times -241.8\text{ kJ}\right) - \left(-74.81\text{ kJ}\right) = -802.31\text{ kJ}\left(/\text{molCH}_4\right)$$

$C_2H_6(g) + \frac{7}{2}O_2(g) \rightarrow 2\ CO_2(g) + 3\ H_2O(g)$

$$\Delta H^{C_2H_6}_{\text{combustion}} = (\left(2\times -393.5\text{ kJ}\right) + \left(3\times -241.8\text{ kJ}\right) - \left(-84.68\text{ kJ}\right) = -1427.7\text{ kJ}\left(/\text{molC}_2\text{H}_6\right)$$

$C_3H_8(g) + 5\ O_2(g) \rightarrow 3\ CO_2(g) + 4\ H_2O(g)$

$$\Delta H^{C_3H_8}_{\text{combustion}} = (\left(3\times -393.5\text{ kJ}\right) + \left(4\times -241.8\text{ kJ}\right) - \left(-103.8\text{ kJ}\right) = -2043.9\text{ kJ}\left(/\text{molC}_2\text{H}_6\right)$$

2) Calculate the total moles of gas and 3) individual moles of each fuel.

V = 385 L T = 22.6°C = 295.8 K P = 739 mmHg = 0.9724 atm

$$n = \frac{PV}{RT} = \frac{\left(0.9724\text{ atm}\right)\left(385\text{ L}\right)}{\left(0.08206\,\frac{\text{L·atm}}{\text{mol·K}}\right)\left(295.8\text{ K}\right)} = 15.43\text{ mol gas}$$

$$n_{CH_4} = 0.830\times 15.43\text{ mol} = 12.80\text{ mol CH}_4$$

$$n_{C_2H_6} = 0.112\times 15.43\text{ mol} = 1.728\text{ mol C}_2\text{H}_8$$

$$n_{C_3H_8} = 0.058\times 15.43\text{ mol} = 0.8949\text{ mol C}_3\text{H}_8$$

4) Calculate the energy produced by the combustion of each and the total energy produced.

$$\Delta H^{CH_4}_{\text{combustion}} = -802.31\text{ kJ/molCH}_4 \quad q_{CH_4} = n\cdot\Delta H^{CH_4}_{\text{combustion}} = 12.80\text{ mol }\times -802.31\text{ kJ/mol CH}_4 = -10,270\text{ kJ}$$

$$\Delta H^{C_2H_6}_{\text{combustion}} = -1427.7\text{ kJ/molC}_2\text{H}_6 \quad q_{C_2H_6} = n\cdot\Delta H^{C_2H_6}_{\text{combustion}} = 1.728\text{ mol }\times -1427.7\text{ kJ/molC}_2\text{H}_6 = -2,467\text{ kJ}$$

$$\Delta H^{C_3H_8}_{\text{combustion}} = -2043.9\text{ kJ/molC}_2\text{H}_6 \quad q_{C_3H_8} = n\cdot\Delta H^{C_3H_8}_{\text{combustion}} = 0.8949\text{ mol }\times -2043.9\text{ kJ/molC}_2\text{H}_6 = -1829\text{ kJ}$$

$$q_{\text{total}} = -14,600\text{ kJ}$$