Gas Laws IV
Kinetic-Molecular Theory

1. What is the root-mean-squared velocity of nitrogen gas at 25.0°C?

$$M_{N_2} = 28.013\, {}^{g}\!/_{mol} = 0.028013\, {}^{kg}\!/_{mol}$$
$$T = 298.2 \text{ K}$$
$$u_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.314 \frac{J}{mol\cdot K})(298.2 \text{ K})}{0.028013\, {}^{kg}\!/_{mol}}} = 515.3\, {}^{m}\!/_{s}$$

2. What is average kinetic energy of the nitrogen molecules at 25.0°C?

$$\overline{KE} = \tfrac{3}{2} RT = \tfrac{3}{2}(8.314 \tfrac{J}{mol\cdot K})(298.2 \text{ K}) = 3719\, {}^{J}\!/_{mol}$$
or...
$$\overline{KE} = \tfrac{1}{2} mu^2 = \tfrac{1}{2}(0.028013\, {}^{kg}\!/_{mol})(515.3\, {}^{m}\!/_{s})^2 = 3719\, {}^{J}\!/_{mol}$$

3. What is the average kinetic energy (in J/mol) of SF_6 at 25.0°C? What is the average velocity of SF_6 at this temperature?

$$\overline{KE} = 3719\, {}^{J}\!/_{mol} = \tfrac{1}{2} mu^2$$
$$u = \sqrt{\frac{2\overline{KE}}{M}} = \sqrt{\frac{2(3719\, {}^{J}\!/_{mol})}{0.14606\, {}^{kg}\!/_{mol}}} = 225.7\, {}^{m}\!/_{s}$$

4. In a 1-meter-long time-of-flight (TOF) mass spectrometer, a helium ion traversed from the ion source to the ion detector in 12.25 ms. A gas of unknown identity traversed the TOF tube in 188 ms. What is the molar mass of the unknown gas?

$$u = \frac{d}{t} = \sqrt{\frac{3RT}{M}} \quad \text{where } t \text{ is time to travel that distance, } d$$

$$\frac{d}{t_{\text{He}}} = \sqrt{\frac{3RT}{M_{\text{He}}}} \quad \text{and} \quad \frac{d}{t_{\text{UK}}} = \sqrt{\frac{3RT}{M_{\text{UK}}}} \quad \text{Ratio the equations to each other to eliminate constants...}$$

$$\frac{d/t_{\text{He}}}{d/t_{\text{UK}}} = \frac{\sqrt{\frac{3RT}{M_{\text{He}}}}}{\sqrt{\frac{3RT}{M_{\text{UK}}}}} \qquad \frac{t_{\text{UK}}}{t_{\text{He}}} = \sqrt{\frac{M_{\text{UK}}}{M_{\text{He}}}} \qquad M_{\text{UK}} = M_{\text{He}}\left(\frac{t_{\text{UK}}}{t_{\text{He}}}\right)^2 = 4.003\,\text{g/mol}\left(\frac{188\text{ ms}}{12.25\text{ ms}}\right)^2 = 943\,\text{g/mol}$$

5. The escape velocity of a object from Earth's gravitational field is about 25,000 mi/h. In units of miles-per-hour, what is the average velocity of helium at 0°C?

$$u_{\text{He}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.314\,\frac{\text{J}}{\text{mol·K}})(273.2\text{ K})}{0.004003\,\text{g/mol}}} = 1304.7\,\frac{\text{m}}{\text{s}}$$

$$u_{\text{He}} = 1304.7\,\frac{\text{m}}{\text{s}} \times \frac{1\text{ mi}}{1609\text{ m}} \times 3600\,\frac{\text{s}}{\text{h}} = 2919\,\text{mi/h}$$

6. At what temperature does the velocity of a helium atom exceed the escape velocity of the Earth?

$$u = 25{,}000\,\text{mi/h} \times 1609\,\text{m/mi} \times \frac{1\text{ h}}{3600\text{ s}} = 11{,}173\,\text{m/s}$$

$$u^2 = \frac{3RT}{M} \qquad T = \frac{u^2 M}{3R} = \frac{(11{,}173\,\text{m/s})^2(0.004003\,\text{kg/mol})}{3(8.314\,\frac{\text{J}}{\text{mol·K}})} = 20{,}000\text{ K}$$