Stellar Properties
Binary Systems

1. Mizar, in the Alcor-Mizar binary system of Ursa Major, is actually a spectroscopic binary. The binary can not be resolved except, perhaps, in the largest telescopes but the absorption line spectrum of the star is alternately red shifted, unshifted, then blue shifted over a period of 4 days. Accurate measurements of the red- and blue-shifts of the H_α line (656.455 nm in the laboratory) shows that it shifts to 656.761 nm then, 4 days later, shifts to 656.149 nm.

   What is the velocity at which the companion star orbits relative to the primary star?

$$\frac{\Delta\lambda}{\lambda_o} = \frac{v}{c}$$

$$\Delta\lambda_{\text{blueshift}} = 656.149\text{ nm} - 656.455\text{ nm} = \text{-}0.306\text{ nm (approaching)}$$

$$\Delta\lambda_{\text{redshift}} = 656.761\text{ nm} - 656.455\text{ nm} = +0.306\text{ nm (receeding)}$$

$$v = 3.00\times10^{8}\,{}^{\text{m}}\!/_{\text{s}}\left(\frac{0.306\text{ nm}}{656.455\text{ nm}}\right) = 1.398\times10^{5}\,{}^{\text{m}}\!/_{\text{s}} = 139.8\,{}^{\text{km}}\!/_{\text{s}}$$

2. Albireo, the "beak" star in the constellation Cygnus the Swan, is arguably the most beautiful binary star system in the Earth's sky. The system is 380 ly away and the stars are separated by 34 arcsec. The current best approximation of the orbital period of the binary system is 7300 years. What is the combined mass of the binary system?

$$d = 380\text{ ly}\times9.461\times10^{15}\,{}^{\text{m}}\!/_{\text{ly}} = 3.60\times10^{18}\text{ m}$$

$$\alpha = 3.4''$$

$$D = \frac{\alpha d}{206,265''} = \frac{(34'')(3.60\times10^{18}\text{ m})}{206,265''} = 5.93\times10^{14}\text{ m}$$

$$P = 7300\text{ y}\times365.25\,{}^{\text{d}}\!/_{\text{y}}\times24\,{}^{\text{h}}\!/_{\text{d}}\times3600\,{}^{\text{s}}\!/_{\text{h}} = 2.30\times10^{11}\text{ s}$$

$$P^2 = \left(\frac{4\pi^2}{G(m_{\text{Total}})}\right)a^3$$

$$m_{\text{Total}} = \left(\frac{4\pi^2}{GP^2}\right)a^3 = \left(\frac{4\pi^2}{(6.6726\times10^{-11}\,{}^{\text{Nm}^2}\!/_{\text{kg}^2})(2.30\times10^{11}\text{s})^2}\right)(5.93\times10^{14}\text{ m})^3$$

$$m_{\text{Total}} = 2.32\times10^{33}\text{ kg}$$