Electromagnetic Radiation

The Inverse-Square Law

11. What is the brightness of the Sun (in W/m^2) when viewed from Earth? Helpful (but, perhaps, not needed) information is found in the table.

Sun	
Surface Temperature	5,800 K
$R_\odot$	6.96 x 10^8 m
$F_\odot$	6.33 x 10^7 W/m^2
$L_\odot$	3.85 x 10^{26} W
$d_{\text{Earth to Sun}}$	1.496 x 10^{11} m

Brightness is simply flux.

$b = \frac{L}{4\pi d^2}$ (inverse-square law)

$b = \frac{3.85\times10^{26}\,\text{W}}{4\pi(1.496\times10^{11}\,\text{m})^2} = 1369\,{}^{\text{W}}\!/_{\text{m}^2}$ (which is same as solar constant)

12. A car's headlights, when viewed from 1.6 km, had a measured flux of 0.004 W/m^2. What is the flux of the headlights when measured at 1.0 m?

$b_{\text{close}} = \frac{L}{4\pi d_{\text{close}}^2}$ and $b_{\text{far}} = \frac{L}{4\pi d_{\text{far}}^2}$

Luminosity of the headlight is independent of location of observer.

Set the two equations equal to each other...

$\frac{b_{\text{close}}}{b_{\text{far}}} = \frac{d_{\text{far}}^2}{d_{\text{close}}^2}$ so... $b_{\text{close}} = b_{\text{far}} \times \frac{d_{\text{far}}^2}{d_{\text{close}}^2} = 0.004\,{}^{\text{W}}\!/_{\text{m}^2} \times \frac{(1600\text{m})^2}{(1\text{ m})^2} = 10,240\,{}^{\text{W}}\!/_{\text{m}^2}$

This sounds like a lot of power, but consider a headlight is only about 20 x 15 cm (0.03 m^2) so the total luminosity is

$L = 10,240\,{}^{\text{W}}\!/_{\text{m}^2} \times 0.03\text{ m}^2 = 307\text{ W}$ *which is not an excessively bright light bulb.*

13. Even though Sirius (α-CMa) is the brightest star in the Earth's sky, its photometric brightness (flux) is only 1.22 x 10^{-7} W/m^2 compared to 1370 W/m^2 for the sun, measured at the Earth's surface. The distance to Sirius is 8.61 ly. From this and other data found in the appendix or at the instructor's website, calculate the luminosity of Sirius (in W). (Incidentally, this is one of the important roles of the so-called *standard candle* for distance measurements.)

$b_* = \frac{L_*}{4\pi d_*^2}$ and $b_\odot = \frac{L_\odot}{4\pi d_\odot^2}$

setting the equations equal to each other...

$\frac{b_*}{b_\odot} = \frac{L_*}{L_\odot}\frac{d_\odot^2}{d_*^2}$ so... $L_* = L_\odot \frac{b_*}{b_\odot}\frac{d_*^2}{d_\odot^2}$

$d_* = (8.61\text{ ly})(9.461\times10^{15}\,\text{m}) = 8.146\times10^{16}\,\text{m}$

$L_* = 3.85\times10^{26}\,\text{W}\,\frac{1.22\times10^{-7}\,{}^{\text{W}}\!/_{\text{m}^2}}{1370\,{}^{\text{W}}\!/_{\text{m}^2}}\frac{(8.146\times10^{16}\text{m})^2}{(1.496\times10^{11}\text{m})^2} = 1.02\times10^{28}\,\text{W}$ (26$\times$ the luminosity of the Sun)