Electromagnetic Radiation
Blackbody Emission, Luminosity, Brightness, Flux

5. What is the wavelength of maximum emission, λ_{max}, for a blackbody radiator which has a temperature of 1000 K?

$$\lambda_{max} = \frac{0.0029}{T} \text{ (Wien's Law)} \qquad \lambda_{max} = \frac{0.0029}{1000\text{ K}} = 2.9\times10^{-6}\text{ m} = 2,900\text{ nm (IR)}$$

6. λ_{max} of the Sun is 500 nm (green). What is the apparent surface temperature of the Sun?

$$\lambda_{max} = \frac{0.0029}{T} \text{ (Wien's Law)}$$

$$\lambda_{max} = 500\text{ nm}\times\frac{1\text{ m}}{10^9\text{nm}} = 5.00\times10^{-7}\text{ m}$$

$$T = \frac{0.0029}{\lambda_{max}} = \frac{0.0029}{5.00\times10^{-7}\text{ m}} = 5,800\text{ K}$$

7. Sirius (α-CMa) has a surface temperature of ~10,000 K. What is the wavelength of maximum emission, λ_{max}, for Sirius? What color would Sirius be expected to appear?

$$\lambda_{max} = \frac{0.0029}{T} \text{ (Wien's Law)} \qquad \lambda_{max} = \frac{0.0029}{10,000\text{ K}} = 2.9\times10^{-7}\text{ m} = 290\text{ nm (UV)}$$

The spectrum of Sirius will be rich in the blue end of the visible light continuum. Thus, it will look blue-white.

8. What is the λ_{max} and the electromagnetic radiation flux (energy emitted per square meter) of an electric stove coil at room temperature (25°C)?

$$\lambda_{max} = \frac{0.0029}{T} \text{ (Wien's Law)} \qquad T = 25°\text{C} + 273 = 298\text{ K}$$

$$\lambda_{max} = \frac{0.0029}{298\text{ K}} = 9.73\times10^{-6}\text{ m} = 9,730\text{ nm (IR)}$$

$$F = \sigma T^4 \text{ (Stephan-Boltzmann Law)} \quad \sigma = 5.67\times10^{-8}\ \text{W}/\text{m}^2\text{K}^4$$

$$F = \left(5.67\times10^{-8}\ \text{W}/\text{m}^2\text{K}^4\right)(298\text{ K})^4 = 447\ \text{W}/\text{m}^2$$

What is the λ_{max} and the electromagnetic radiation flux of the same electric stove coil at 1000°C?

$$T = 1000°\text{C} + 273 = 1273\text{ K}$$

$$\lambda_{max} = \frac{0.0029}{1273\text{ K}} = 2.28\times10^{-6}\text{ m} = 2280\text{ nm (IR)}$$

$$F = \sigma T^4 \text{ (Stephan-Boltzmann Law)}$$

$$F = \left(5.67\times10^{-8}\ \text{W}/\text{m}^2\text{K}^4\right)(1273\text{ K})^4 = 149,000\ \text{W}/\text{m}^2$$

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9. At the Earth's surface, the flux due to the Sun is 1370 W/m^2 (known as the solar constant). What is the luminosity (total energy radiated) of the Sun?

The luminosity of the Sun is measured at 1 AU (1.496 x 10^{11} m).

$$L_\odot = F_\odot \times \text{Area} = F_\odot \times 4\pi R_\odot^2$$

$$L_\odot = 1370\ {}^{\text{W}}\!/_{\text{m}^2} \times 4\pi(1.496\times10^{11}\ \text{m})^2 = 3.85\times10^{26}\ \text{W}$$

The unasked question: What is the flux at the Sun's surface? ($R_\odot$ = 6.96 x 10^8 m)

$$L_\odot = F_\odot \times \text{Area} = F_\odot \times 4\pi R_\odot^2 \quad \text{so...}$$

$$F_\odot = \frac{L_\odot}{4\pi R_\odot^2}$$

$$F_\odot = \frac{3.85\times10^{26}\ \text{W}}{4\pi(6.96\times10^{8}\ \text{m})^2} = 6.33\times10^{7}\ {}^{\text{W}}\!/_{\text{m}^2}$$

10. Use the following data to determine the radius of Sirius (α-CMa)

Sun	
Surface Temperature	5,800 K
$R_\odot$	6.96 x 10^8 m
$F_\odot$ (at surface)	6.33 x 10^7 W/m^2
$L_\odot$	3.85 x 10^{26} W
Sirius	
Surface Temperature	10,000 K
L_*	26$L_\odot$ = 1.00 x 10^{28} W

There are two approaches to this problem.

Approach 1:

$$F_* = \sigma T_*^4 \quad \text{(Stephan-Boltzmann Law)} \quad \sigma = 5.67\times10^{-8}\ {}^{\text{W}}\!/_{\text{m}^2\text{K}^4}$$

$$F_* = \left(5.67\times10^{-8}\ {}^{\text{W}}\!/_{\text{m}^2\text{K}^4}\right)(10000\ \text{K})^4 = 5.67\times10^{8}\ {}^{\text{W}}\!/_{\text{m}^2}$$

$$F_* = \frac{L_*}{4\pi R_*^2} \quad \text{so...} \quad R_* = \sqrt{\frac{L_*}{4\pi F_*}} = \sqrt{\frac{1.00\times10^{28}\ \text{W}}{4\pi(5.67\times10^{8}\ {}^{\text{W}}\!/_{\text{m}^2})^2}} = 1.18\times10^{9}\ \text{m} \quad (1.7\times \text{ larger than the Sun})$$

Approach 2: (Eliminates the need to know σ)

$$F_* = \sigma T_*^4 \quad \text{and} \quad F_\odot = \sigma T_\odot^4 \quad \text{so...} \quad \frac{F_*}{F_\odot} = \frac{T_*^4}{T_\odot^4}$$

$$F_* = \frac{(10000\ \text{K})^4}{(5800\ \text{K})^4} \times 6.33\times10^{7}\ {}^{\text{W}}\!/_{\text{m}^2} = 5.59\times10^{8}\ {}^{\text{W}}\!/_{\text{m}^2} \quad \text{(for Sirius)}$$

$$F_* = \frac{L_*}{4\pi R_*^2} \quad \text{so...} \quad R_* = \sqrt{\frac{L_*}{4\pi F_*}} = \sqrt{\frac{1.00\times10^{28}\ \text{W}}{4\pi(5.59\times10^{8}\ {}^{\text{W}}\!/_{\text{m}^2})}} = 1.19\times10^{9}\ \text{m}$$