Combining Percentage Composition, Concentration, and Equilibrium

Problem 16-33

A particular vinegar is found to contain 5.7% acetic acid, $HC_2H_3O_2$, by mass. What mass of the vinegar should be diluted with water to produce 0.750 L of a solution with pH = 4.52?

Solution:

This integrative problem requires quite a bit of information management to get to an answer.

Let's start by getting the one thing we really know, $[H_3O^+]$:

$$[H_3O^+] = 10^{-pH} = 10^{-4.52} = 3.02\times10^{-5}\ \text{M}$$

Now, we can calculate the $[CH_3CO_2H]$ necessary to obtain that $[H_3O+]$:

$$CH_3CO_2H\ +\ H_2O \rightleftharpoons CH_3CO_2^-\ +\ H_3O^+$$

$$K_a = 1.75\times10^{-5} = \frac{[H_3O^+][CH_3CO_2^-]}{[CH_3CO_2H]}$$

But, we know that $[H_3O^+] = [CH_3CO_2^-]$ by stoichiometry, so...

$$1.75\times10^{-5} = \frac{\left(3.02\times10^{-5}\right)^2}{C_{CH_3CO_2H} - 3.02\times10^{-5}}$$ Do the algebra,

$$C_{CH_3CO_2H} = 8.23\times10^{-5}\ \text{M}\ CH_3CO_2H$$

The problem stipulates 0.750 L of the diluted solution, so we can calculate the quantity, in moles, of acetic acid in the solution:

$$n_{CH_3CO_2H} = 8.23\times10^{-5}\ \text{M}\times 0.750\ \text{L} = 6.17\times10^{-5}\ \text{mol}\ CH_3CO_2H$$

We get that acetic acid from the 5.7%$^{w}/_{w}$ solution of vinegar. We need only to calculate the mass of 5.7% acetic acid that contains that number of moles of acetic acid:

$$m_{CH_3CO_2H} = \frac{6.17\times10^{-5}\ \text{mol}\ CH_3CO_2H}{\left(0.057\ \frac{\text{g}\ CH_3CO_2H}{\text{g vinegar}} \Big/ 60.05\ \frac{\text{g}}{\text{mol}}\right)} = 0.065\ \text{g vinegar}$$