Solubility Equilibrium and Recrystallization

Problem 13-42

One way to recrystallize a solute from a solution is to change the temperature. Another way is to evaporate solvent from the solution. A 335-g sample of a saturated solution of $KNO_3(s)$ in water is prepared at 25.0°C. If 55 g H_2O is evaporated from the solution at the same time as the temperature is reduced from 25.0 to 0.0 °C, what mass of $KNO_3(s)$ will recrystallize? (Refer to Figure 13-8.)

Solution:

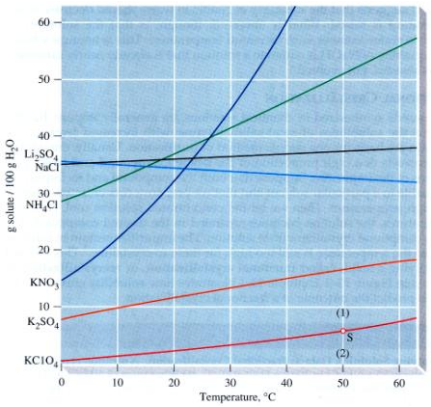

Figure 13-8 is necessary to get the solubility of KNO_3 at both temperatures (shown on the graph).

At 0°C, s_{KNO_3} = 15 g KNO_3/100 g H_2O
At 25°C, s_{KNO_3} = 38 g KNO_3/100 g H_2O

The solution to the problem can be done in 2 steps: 1) evaporate the water and calculate the amount of KNO_3 that precipitates and 2) reduce the temperature and calculate again. There are several ways to achieve the same results. It's best to convert the mass of sample to mass of water in the sample.

s_{KNO_3} = 38 g KNO_3/100 g H_2O can be rewritten as s_{KNO_3} = 38 g KNO_3/138 g solution

$$s_{KNO_3} = \frac{38 \text{ g } KNO_3}{100 \text{ g } H_2O + 38 \text{ g } KNO_3} = 0.275 \frac{\text{g } KNO_3}{\text{g soln}}$$

or as

$$c_{H_2O} = 100 \text{ g } H_2O/138 \text{ g solution}$$

So, the mass of KNO_3 in the 355 g of solution is

$$m_{KNO_3} = 0.275 \frac{\text{g } KNO_3}{\text{g soln}} \times 335 \text{ g soln} = 92.3 \text{ g } KNO_3 \text{ dissolved at 25°C}$$

and the mass of water in the 355 g of solution is $m_{H_2O} = \frac{100 \text{ g } H_2O}{138 \text{ g soln}} \times 335 \text{ g soln} = 242.8 \text{ g } H_2O$

Now, remove 55 g of H_2O, lower the temperature, and recalculate the mass of dissolved KNO_3:

$$m_{KNO_3} = 187.8 \text{ g } H_2O \times \frac{15 \text{ g } KNO_3}{100 \text{ g } H_2O} = 28.2 \text{ g } KNO_3 \text{ dissolved at 0°C}$$

Almost done: calculate the amount of KNO_3 that precipitates after the temperature change from the amounts remaining in solution

$$m_{KNO_3 \text{ ppt}} = 92.3 \text{ g originally} - 28.2 \text{ g remaining} = 64.1 \text{ g } KNO_3 \text{ ppts}$$