Heat Balance and Phase Changes

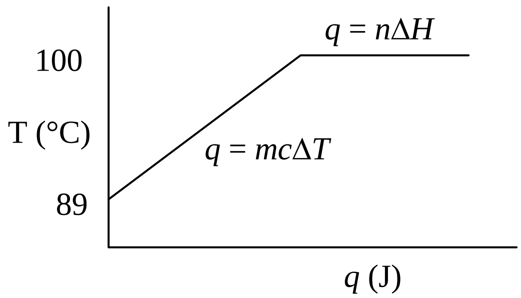
Problem 12-12

A 50.0-g piece of iron at 152°C is dropped into 20.0 g H_2O(l) at 89°C in an open, thermally insulated container. How much water would you expect to vaporize, assuming no water splashes out? The specific heats of iron and water are 0.45 and 4.21 J $g^{-1}°C^{-1}$, respectively, and ΔH_{vap} = 40.7 kJ mol^{-1} H_2O.

Solution

Notice that the value for the specific heat of water is 4.21 J $g^{-1}°C^{-1}$ here. This illustrates the slight temperature dependence on specific heat that we often ignore.

If you draw a heating curve for water in this problem, it might look something like this:



The maximum temperature the water can reach is 100°C, at which point a phase change will occur. The temperature of the iron, then, goes to 100°C as well. If you just write

$$q_{Fe} + q_{H_2O} = 0 = m_{Fe}c_{Fe}\Delta T_{Fe} + m_{H_2O}c_{H_2O}\Delta T_{H_2O}$$

You discover that the maximum temperature obtained is more than 100°C, which is impossible. So, the correct energy balance must be

$$m_{Fe}c_{Fe}\Delta T_{Fe} + m_{H_2O}c_{H_2O}\Delta T_{H_2O} + n_{H_2O\text{ vaporized}}\Delta H_{vap} = 0$$

Now, let's fill in the variables of the things we know and calculate what we don't:

$$0 = (50.0\text{ g})\left(0.45\tfrac{\text{J}}{\text{g}\cdot°\text{C}}\right)(100°\text{C} - 152°\text{C}) + (20.0\text{ g})\left(4.21\tfrac{\text{J}}{\text{g}\cdot°\text{C}}\right)(100°\text{C} - 89°\text{C}) + n_{H_2O\text{ vaporized}}\left(40.7\times10^3\tfrac{\text{J}}{\text{mol}}\right)$$

$$0 = -1170\text{ J} + 926.2\text{ J} + n_{H_2O\text{ vaporized}}\left(40.7\times10^3\tfrac{\text{J}}{\text{mol}}\right)$$

$$n_{H_2O\text{ vaporized}}\left(40.7\times10^3\tfrac{\text{J}}{\text{mol}}\right) = 243.8\text{ J}$$

$$n_{H_2O\text{ vaporized}} = 0.00599\text{ mol H}_2\text{O}$$

And finish by calculating the mass of water vaporized…

$$m_{H_2O} = 0.00599\text{ mol}\times18.015\tfrac{\text{g}}{\text{mol}} = 0.108\text{ g}$$