Kinetic Molecular Theory and Quantum Mechanics: An Integrative Problem

Problem 8-103

Certain metal compounds impart colors to flames – sodium compounds, yellow; lithium, red; barium, green – and flame tests can be used to detect these elements. (**a**) At a flame temperature of 800°C, can collisions between gaseous atoms with average kinetic energies supply the energy require for the emission of visible light? (**b**) If not, how do you account for the excitation energy?

Solution:

This integrative problem requires 1) the calculation of kinetic energy from the principles of kinetic molecular theory and 2) the estimation of the lowest energy of visible light.

Start with the calculation of kinetic energy per atom. Recall that regardless of the gas the kinetic energy of all gases is the same at the same temperature.

$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ and $\overline{KE} = \frac{1}{2} m v_{\text{rms}}^2$ We can modify the KE equation to use molar mass, M, by dividing by Avogadro's number: $\overline{KE} = \frac{1}{2} \frac{M}{6.022 \times 10^{23} \frac{\text{atoms}}{\text{mol}}} v_{\text{rms}}^2$

Substitute the first equation into the second for simplicity. This isn't necessary but is shortens the problem by one step

$$\overline{KE} = \frac{1}{2} \frac{M}{6.022 \times 10^{23} \frac{\text{atoms}}{\text{mol}}} \left(\sqrt{\frac{3RT}{M}} \right)^2 = \frac{3}{2} \times \frac{RT}{6.022 \times 10^{23} \frac{\text{atoms}}{mol}}$$

Now fill in some numbers:

$$\overline{KE} = \frac{3}{2} \times \frac{\left(8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right)\left(1073 \text{ K}\right)}{6.022 \times 10^{23} \frac{\text{atoms}}{mol}} = 2.22 \times 10^{20} \text{J(/atom)}$$

Now, calculate the energy of, say, red light (the lowest visible light energy). 700 nm is picked here for convenience.

$$E = \frac{hc}{\lambda} = \frac{\left(6.626 \times 10^{-34} \text{J} \cdot \text{s}\right)\left(3.00 \times 10^{8} \frac{\text{m}}{\text{s}}\right)}{7 \times 10^{-7} \text{ m}} = 2.8 \times 10^{-19} \text{J(/photon)}$$

This is about 10 times greater energy than that supplied by the collision of the atoms. Hardly enough energy to excite the ground state atom to emit even red, let alone yellow or green which are higher energy.

But, recall that not all atoms have the average velocity. Some are faster, much faster than average thus supplying the necessary kinetic energy to get *some* of the atoms into excited states.