Mass, Atoms, Moles, Density: An Integrative Problem

The German chemist Fritz Haber proposed paying off the reparations imposed against Germany in World War I by extracting gold from seawater. Given that (1) the amount of reparations was 28.8 billion dollars, (2) the value of gold at the time was about \$21.25 per troy ounce (1 troy ounce = 31.103 g), and (3) gold occurs in seawater to the extent of 4.67×10^{17} atoms per ton of seawater (1 ton = 2000 lb), how many cubic kilometers of seawater would have had to be processed to obtain the required amount of gold? Assume that the density of seawater is1.03 g/cm^3.

Solution:

Start by determining the mass of Au necessary for the reparations:

$$m = 28.8\times10^{9}\ \text{dollars} \Big/ 21.25\,\tfrac{\text{dollars}}{\text{troy}} = 1.3553\times10^{9}\ \text{troy Au}$$

$$m = 1.3553\times10^{9}\ \text{troy Au}\times\frac{31.103\ \text{g}}{\text{troy}} = 4.2151\times10^{10}\ \text{g Au}$$

Calculate the concentration of Au in seawater:

$$\text{C}=\frac{4.67\times10^{17}\text{atoms}\times\dfrac{1\ \text{mol}}{6.022\times10^{23}\text{atoms}}\times196.97\,\tfrac{\text{g}}{\text{mol}}}{\text{ton seawater}} = 1.5275\times10^{-4}\,\tfrac{\text{g Au}}{\text{ton seawater}}$$

The mass of seawater necessary to get the require amount of Au is

$$m_{\text{seawater}} = 4.2151\times10^{10}\ \text{g Au}\times\frac{\text{ton seawater}}{1.5275\times10^{-4}\text{g Au}} = 2.7597\times10^{14}\ \text{ton seawater}$$

The volume of seawater is calculated using density:

$$V_{\text{seawater}} = 2.7597\times10^{14}\ \text{ton seawater}\times2000\,\tfrac{\text{lb}}{\text{ton}}\times453.6\,\tfrac{\text{g}}{\text{lb}}\times\frac{1\ \text{cm}^3\ \text{seawater}}{1.03\ \text{g seawater}}\times\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3\times\left(\frac{1\ \text{km}}{1000\ \text{m}}\right)^3$$

$$= 2.43\times10^{5}\,\text{km}^3$$