Dimensional Analysis
A More Complex Problem

For a touch of reality (or absurdity) with particular appeal for the "trivia buff', try your hand at the following problem involving *dimensional analysis.* The problem is based on data given in the *Guinness Book of World Records.*

Problem:
The Amazon River has the greatest flow of any river in the world, discharging an average of 4,200,000 (2 significant figures) cubic feet of water per second into the Atlantic Ocean. If there are 1.48×10^{18} tons of water on Earth, how many years are required for the Amazon's flow to equal the Earth's water supply? (Assume a density of $1.0\ g/cm^3$ for all water.)

Get rid of words and minimize the problem to "haves" and "needs":

Have:

Discharge rate = $R = 4.2 \times 10^6\ ft^3/s$

$m_{\text{water on Earth}} = 1.48 \times 10^{18}$ tons

$d_{\text{water}} = 1.0\ g/cm^3$

Problem:

What time, t, is required discharge 1.48×10^{18} tons water from Amazon River?

Need:
Conversions:
ft^3 to tons (or *vice versa*)
s to years

Don't have any of these so derive them from known conversions:

1 ton = 2000 lb
453.6 g = 1 lb

12 in = 1 ft
2.54 cm = 1 in

60 s = 1 min; 60 min = 1 h; 3600 s = 1 h
24 h = 1 d
365 d = 1 y

Solve the problem: (shown is just one way to do it)

$$R = 4.2\times10^{6}\ \text{ft}^3/\cancel{\text{s}} \times \frac{3600\ \cancel{\text{s}}}{1\ \cancel{\text{h}}} \times \frac{24\ \cancel{\text{h}}}{1\ \cancel{\text{d}}} \times \frac{365\ \cancel{\text{d}}}{1\ \text{y}} \times \left(\frac{12\ \text{in}}{1\ \text{ft}}\right)^3 \times \left(\frac{2.54\ \text{cm}}{1\ \text{in}}\right)^3$$

$$R = 3.751\times10^{18}\ \text{cm}^3/\text{y}$$

Ok, this is the discharge rate, the magnitude of which is nearly incomprehensible. Now, calculate the volume of water over the Earth in cubic centimeters...

$$V = 1.48\times10^{18}\ \text{tons} \times \frac{2000\ \text{lb}}{1\ \text{ton}} \times \frac{453.6\ \text{g}}{1\ \text{lb}} \times \frac{1\ \text{cm}^3}{1\ \text{g}}$$

$$V = 1.343\times10^{24}\ \text{cm}^3\ \text{(another obscenely large number!)}$$

Last step... use dimensional analysis one more time to arrange values to give time.

$$t = 1.343\times10^{24}\ \text{cm}^3 \times \frac{1\ \text{y}}{3.751\times10^{18}\ \text{cm}^3} = 358,000\ \text{y} = \boxed{360,000\ \text{y}}\ \text{(to 2 SF)}$$

The important thing to understand is that there are other approaches to solving this problem that will give identical results. Another aspect of this to realize is that you never once had to memorize a formula to solve the problem. Be careful, though, dimensional analysis is a crutch that is not very useful if used without understanding and thinking.