Dimensional Analysis: Density and Concentration

Problem 1-73

Magnesium occurs in seawater to the extent of 1.4 g magnesium per kilogram of seawater. What volume of seawater, in cubic meters, would have to be processed to produce 1.00×10^5 tons of magnesium (1 ton = 2000 lb)? Assume a density of 1.025 g/mL for seawater.

Solution:

First, eliminate words and define the problem:

$$C_{\text{Mg in seawater}} = 1.4 \frac{\text{g Mg}}{\text{kg seawater}} \qquad m_{\text{Mg produced}} = 10^5 \text{ tons} \qquad d_{\text{seawater}} = 1.025 \, \text{g/mL}$$

Conversions:

1 ton = 2000 lb

Problem:

$V_{\text{seawater cont'g } 10^5 \text{ tons Mg}} = ?$

The problem is pretty straight-forward except none of the units are consistent. Decide what units you want to use and it becomes a mass-volume-density problem.

Doing the problem using the density equation, $d = \frac{m}{V}$, then $V = \frac{m}{d}$

Let's first calculate the mass of Mg produced:

$$m_{\text{Mg}} = 10^5 \cancel{\text{tons}} \times 2000 \frac{\cancel{\text{lb}}}{\cancel{\text{ton}}} \times 453.6 \frac{\text{g}}{\cancel{\text{lb}}} = 9.072 \times 10^{10} \text{ g Mg} \quad \text{(with 1 significant figure)}$$

Now calculate the mass of seawater which contains this amount of Mg:

$$m_{\text{seawater}} = 9.072 \times 10^{10} \text{ g Mg} \times \frac{1 \text{ kg seawater}}{1.4 \text{ g Mg}} = 6.480 \times 10^{10} \text{ kg seawater}$$

Finally, calculate the volume of seawater using density. Since the problem asks for cubic meters in the end, don't forget the conversion $\text{mL} \longrightarrow \text{cm}^3 \longrightarrow \text{m}^3$.

$$V_{\text{seawater}} = 6.480 \times 10^{10} \text{ kg seawater} \times \left(\frac{1 \text{ cm}^3 \text{ seawater}}{1.025 \text{ g seawater}} \times \frac{1000 \text{ g}}{1 \text{ kg}} \right) \times \left(\frac{1 \text{ m}}{100 \text{ cm}} \right)^3 = 6.32 \times 10^7 \text{ m}^3 \text{ seawater}$$

$$V_{\text{seawater}} = 6 \times 10^7 \text{ m}^3 \text{ seawater} \quad \text{(1 significant figure)}$$