Integration of Percentage Concentration, Density, and Dilution

Problem 1-69

A solution used to chlorinate a home swimming pool contains 7% chlorine by mass. An ideal chlorine level for the pool is one part per million (1 ppm). (Think of 1 ppm as being 1 g chlorine per million grams of water.) If you assume densities of 1.10 g/mL for the chlorine solution and 1.00 g/mL for the swimming pool water, what volume of the chlorine solution, in liters, is required to produce a chlorine level of 1 ppm in an 18,000-gallon swimming pool?

Solution:

This is a fairly challenging integrative problem since it is asking for the amount of a more concentrated solution necessary to make a diluted solution while, at the same time, requiring the analyst to convert volumes and concentrations into compatible units. Split the problem up into bite-sized pieces, separating what is known from what is wanted.

Known:

$$C_{\mathrm{Cl}} = 7\% \,{}^{w}\!/\!_{w} = \frac{7 \text{ g Cl}}{100 \text{ g solution}} \qquad d_{\text{solution}} = 1.10 \, \tfrac{\text{g solution}}{\text{mL solution}}$$

Want:

$$C_{\mathrm{Cl}} = 1 \text{ ppm} = \frac{1 \text{ g Cl}}{10^{6} \text{ g pool water}} \qquad d_{\text{pool water}} = 1.00 \, \tfrac{\text{g pool water}}{\text{mL pool water}}$$

$$V_{\text{pool water}} = 18,000 \text{ gal}$$

The first order of business is to get the volume of the pool water into something similar to the known information, such as milliliters:

1 gal $= 3.7854$ L (look this conversion up)

$$V_{\text{pool water}} = 18,000 \text{ gal} \times \frac{3.7854 \text{ L}}{1 \text{ gal}} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 6.8137 \times 10^{7} \text{ mL pool water}$$

Now, work the problem backwards. We know the volume of the pool so we can calculate the mass of the water in the pool and, with that mass, calculate the amount of chlorine we need in the pool to make the concentration 1 ppm.

$$m_{\text{pool water}} = 6.8137 \times 10^{7} \text{ mL pool water} \times 1.00 \, \tfrac{\text{g pool water}}{\text{mL pool water}} = 6.8137 \times 10^{7} \text{g pool water}$$

Calculate the mass of chlorine in the pool:

$$m_{\mathrm{Cl}} = 6.8137 \times 10^{7} \text{g pool water} \times \frac{1 \text{ g Cl}}{10^{6} \text{ g pool water}} = 68.137 \text{ g Cl}$$

We're very close now. There is 68.137 g Cl in the 18,000 gal pool to give a concentration of 1 ppm. But where did that Cl come from? It came from a volume of 7%$^{w}/_{w}$ Cl solution. So now use the concentration and density of the concentrated Cl solution to work back to volume.

$$m_{\text{solution}} = 68.137 \text{ g Cl} \times \frac{100 \text{ g solution}}{7 \text{ g Cl}} = 973.4 \text{ g solution}$$

But, it's volume we want:

$$V_{\text{solution}} = 973.4 \text{ g solution} \times \frac{1 \text{ mL solution}}{1.10 \text{ g solution}} = 884.9 \text{ mL solution}$$

Working through the correct number of significant figures (1 SF from the 7%)

$$V_{\text{solution}} = 900 \text{ mL solution } (= 9 \times 10^2 \text{ mL solution}) = 0.9 \text{ L}$$